Bank Deposit Mix

and

Aggregate Implications for Financial Stability

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Abstract

This paper analyzes the bank deposit mix and heterogeneous liquidity risk across different sizes of banks, as well as the aggregate implications for financial stability within a model of heterogeneous banks with a deposit market power. I present empirical observations of bank deposit products and composition in the balance sheet across the U.S. bank size distribution. Data shows that banks mix savings and time deposits, with the share of savings deposits increasing as banks grow. In the model, banks engage in liquidity transformation and face liquidity risk due to the potential withdrawal of savings deposits, which incentivizes them to hold liquid assets. Costly but stable time deposits can mitigate withdrawal risk, allowing banks to substitute securities with loans. Withdrawal shock distribution recovered using the model reveals that medium-sized banks are as risky as small banks, despite their significant role in the aggregate asset market. An increased risk of withdrawals in the economy amplifies the banking industry's response to negative net worth shocks, particularly by further reducing the supply of loans. External bank Liquidity provision can alleviate the impact of these shocks showing a distributional effect, with the medium-sized banks benefiting the most from liquidity provision.

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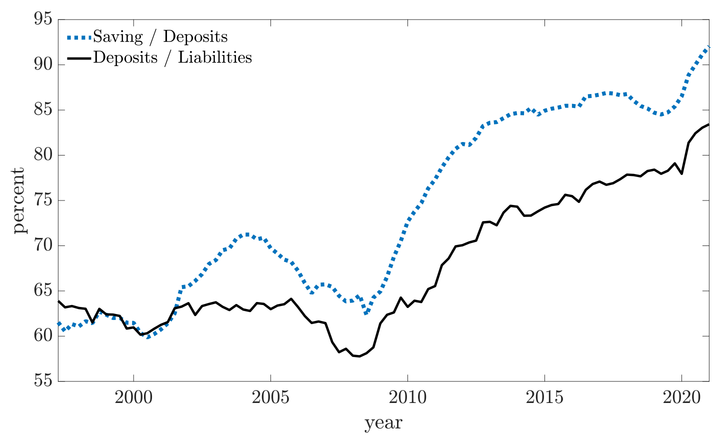
1 Introduction

Banks heavily rely on bank deposits as their primary funding source for liquidity and maturity transformation constituting approximately 70 percent of their liabilities as shown in Figure 1 (solid line). However, these deposits are susceptible to abrupt withdrawals, posing liquidity risk (Diamond and Dybvig (1983)). In response to deposit outflows, banks may need to liquidate assets, potentially resulting in capital losses due to discounted asset sales and foregone interest income. Therefore, the management of liquidity risk with maintaining stable deposit flows is crucial for financial stability and has a significant impact on the aggregate economy.



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1984 Q1 to 2021 Q2. Every aggregation is asset-weighted. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits.

Figure 1: Aggregate deposits in the U.S.

The deposit market has witnessed an increasing trend in the share of savings deposits, achieved by reducing time deposits as in Figure 1 (dotted line)[1]. To understand the consequence of the evolving composition of the bank deposit market, it is crucial to recognize the role of the deposit structure (the mix) for both individual banks and the banking in-

[1] A real-life example of time deposits is a certificate of deposits (CD), and a typical example of demand deposits is a checking account which is mainly used for transactions. Saving deposits include savings accounts and money market deposit accounts (MMDA)

dustry. Given the high concentration of the U.S. banking industry and varying asset and deposit profiles across different bank sizes (Corbae and D'Erasmo (2021), d'Avernas, Eisfeldt, Huang, Stanton, and Wallace (2023)), this paper addresses two key questions: First, How do bank deposit mix and liquidity risk of different sizes of banks impact the financial stability? Second, What are the implications for macro-prudential policies? To answer the previous questions, this paper starts by documenting empirical facts on the U.S. commercial bank's deposit structure and its composition in the balance sheet over bank size distribution. Leveraging data on the U.S. deposit market, we construct a model of heterogeneous banks with liquidity risk and portfolio choice in both assets and liabilities. The model is used to quantitatively study the role of withdrawal risk and heterogeneous liquidity risk in financial stability of the aggregate economy.

The paper examines empirical patterns in the bank deposit structure and its distribution within the U.S. economy, using data from commercial banks' balance sheets and income statements reported in the Call Report. Bank deposits are classified into two main categories: savings deposits and time deposits. During the sample period, banks consistently issued time deposits at higher interest rates compared to savings deposits. In terms of deposit flow, savings deposits exhibit a higher volatility than time deposits. When projecting deposit flows based on bank size distribution, large banks demonstrate a higher degree of stability compared to smaller banks. Smaller banks face larger savings outflows more frequently. This heterogeneity in deposit flows across different asset classes implies that an additional unit of deposits can have a disparate impact on the overall deposit flows. Analyzing the average balance sheet composition across individual banks of different size classes, we find that large banks allocate over 80 percent of their total deposits to savings deposits. Conversely, smaller banks increasingly rely on time deposits. Among the bottom 90 percent of banks, 40 percent of total deposits consist of time deposits. Considering the free withdrawal feature of savings deposits, banks strategically choose a deposit mix, balancing the interest cost of deposits with the stability of funding flows.

I construct a macroeconomic model of banking industry dynamics, focusing on heterogeneous banks and their funding mix choices. The goal is twofold: first, to explain the empirical observations related to bank deposit structures, and second, to analyze the aggregate implications of these structures. In the model, banks use net worth, issue equity, savings, and time deposits to invest in liquid securities and illiquid loans. In the middle of each period, a fraction of savings deposits are withdrawn and the bank must service the withdrawal by re-balancing the asset portfolio. A discount on loans when sold before maturity lets banks choose time deposits to invest in loans and gives an incentive to hold securities to meet withdrawal requests. So they are matching liquidities in assets and deposits. Time deposits are more expensive but free from withdrawal in the middle of each period. Consequently, banks optimize their deposit mix by weighing the cost of deposits against the stability of funding flows.

Liquidity risk, stemming from the withdrawal of savings deposits, directly impacts bank capital. The utilization of assets to fulfill withdrawals, along with discounted asset sales and foregone interest income, results in realized net worth falling short of the maximal net worth that a bank can hypothetically achieve without any withdrawal risk. Notably, significant bank capital losses occur when illiquid loans are sold at a discount or when the risk-free rate is high.

Withdrawal shock is assumed to vary across different size types of banks. Using the model's optimality conditions of banks, we recover the withdrawal shock from the data. Large banks' savings deposits are faced with a smaller and more concentrated withdrawal shock than small banks. Medium-sized banks are exposed to a similar level of withdrawal risk to small banks despite their significant role in explaining aggregate assets. The distribution of withdrawal shock is size-dependent and quantitatively matters as we show in the experiment later.

Using specified functional forms and a set of parameters, we empirically validate our model and quantitatively explore the impact of the bank deposit structure in steady-state comparison experiments. Notably, the model successfully captures qualitative patterns in bank balance sheets, considering both assets and deposits across varying bank sizes. For large banks, withdrawal risk per unit of deposits is diminished, prompting them to finance at lower interest rates by favoring savings deposits. Additionally, a low withdrawal risk allows large banks to hold fewer securities compared to their smaller counterparts. Conversely, small banks prioritize time deposits to mitigate withdrawal risk and allocate more resources to securities, safeguarding their bank capital.

We compare the steady-state outcomes between a baseline economy and one with a low risk-free interest rate to examine the model's mechanisms and understand the aggregate implications of deposit structure. In an environment with a low risk-free rate, a narrowed spread between savings and time deposits lowers the average funding cost of the bank, and the loss in bank capital associated with withdrawal is reduced. However, time deposits within the banking sector decline due to reduced opportunity costs for households holding savings deposits. The rise in the dollar amount of savings deposits exposed to withdrawal risk makes banks demand more liquid assets which crowds out illiquid loans. In another experiment with state-dependent withdrawal risk, we show the medium-sized banks and how much the change in their withdrawal shock distribution affects the aggregate economy. A higher liquidity risk for medium-sized banks reduces a bank's balance sheet. It reduces the share of savings deposits and loans.

To further study the role of deposit mix and heterogeneous bank liquidity in short-run dynamics, I consider the perfect foresight with an adverse aggregate shock to bank net worth. An increase in bank withdrawal risk amplifies the response of the banking sector so it further reduces the loan supply. The presence of withdrawal shock and capital requirement constraint incentivizes banks to exhibit flight-to-liquidity and the loan supply falls. Banks increase the share of savings deposits to lower the funding cost and it makes more deposits are exposed to withdrawal risk. This further reduces the loan supply.

Lastly, I provide a policy implication of the deposit mix and heterogeneous bank liquidity risk. Suppose the central bank provides liquidity when the bank faces large and/or unexpected outflows in liabilities through a discount window. Given the adverse net worth shock, liquidity provision by the central bank accelerates the recovery of the bank's net worth. As the effective withdrawal is reduced, the bank demands fewer securities, and the loan supply does not need to fall as much as the baseline economy. Lowered effective withdrawal risk induces banks to issue more savings deposits and lower the average funding cost. Looking at the response across different size groups, there is a distributional effect of the discount window. Large and medium-sized banks mostly benefited in terms of lowering demand for securities. However, large banks are not very responsive in increasing loan supply from the liquidity provision. Medium-sized banks that are as large as large banks and as risky as small banks are the most responsive and increase the loan supply the most. The model can be used to study banking regulation, especially for liquidity requirements proposed in Basel III. While studying the effect of the regulation on the banking industry and aggregate consequences, size-dependent liquidity risk can extend the understanding of how differently such a policy can affect banks and form aggregate outcomes.

This paper contributes to several strands of macroeconomic literature. First, extensive research has explored the role of banks in macroeconomics. For instance, Gertler and Kiyotaki (2010) investigates the real effect of financial intermediaries during the crisis. Gertler and Kiyotaki (2015) develops a model that delivers an equilibrium bank run over the business cycles. Deposits in banking problems are often assumed as an exogenous process (Corbae and D'Erasmo (2021), Rıos-Rull, Takamura, and Terajima (2020)) or one-period single type of debt (Gertler and Kiyotaki (2015)). In contrast, our paper addresses this gap by endogenizing the deposit market with a realistic deposit mix within banks and deposit market power to analyze macroeconomic implications.

Furthermore, the interplay between realistic deposit structures and market dynamics has been a subject of study in both finance and macro-finance. Jermann and Xiang (2023) models deposit withdrawals for non-maturing saving deposits and studies debt dilution in the context of banking. Supera (2021) which is the most closely related to this paper focuses on the funding mix of banks and explains the decline of firm entry rate by falling time deposits with monetary policy. Drechsler, Savov, and Schnabl (2021) explores deposit market power, revealing that banks with such power respond insensitively to monetary policy regarding deposit rates. Our paper stands apart by modeling heterogeneous banks with funding mix choices, allowing us to analyze the distributional impact of deposit structure within the banking industry and its broader macroeconomic implications.

Thirdly, numerous studies have focused on models emphasizing banking industry dynamics and exploring their aggregate implications across various contexts. Corbae and D'Erasmo (2021) investigates the capital requirement regulation within a concentrated banking industry. Similarly, Rıos-Rull et al. (2020) examines how countercyclical capital requirements impact aggregate dynamics in Hopenhayn (1992) version of the banking model. Dempsey and Faria-e Castro (2022) studies the role of the relationship between borrower and lender and its aggregate consequences. Our paper contributes to this literature by incorporating a realistic deposit structure observed in data into a dynamic model of the banking industry. This approach allows us to study the role of bank deposits in banks' life cycles and explore their aggregate effects. Furthermore, our framework serves as a laboratory for studying the impact of regulations within the banking industry equilibrium, considering the specific characteristics of bank deposit structures.

The subsequent sections of this paper are structured as follows: In Section 2, we present empirical evidence concerning bank deposits and their distribution within the U.S. economy. Following this, Section 3 introduces the model economy. Section 4 analyzes the role of withdrawal risk in bank capital and the household problem to inspect the core implication of the funding mix to the bank and household. Section 5 outlines a quantitative approach for aligning our model with empirical data. In Section 6, we empirically validate the model. Our quantitative analysis, steady-state comparison experiments, is detailed in Section 7. Finally, Section 8 provides concluding remarks.

2 Bank Deposits in the U.S. Economy

This section provides empirical facts on the deposit products and their distribution across U.S. commercial banks. The data used in this analysis is Consolidated Reports of Condition and Income (so-called Call Report) and focuses on U.S. commercial banks located within the 50 states and the District of Columbia. The dataset covers information about the bank's balance sheet and income statement and we restrict our sample periods from 1984 Q1 to 2021 Q2. Here is the summary of the empirical facts:

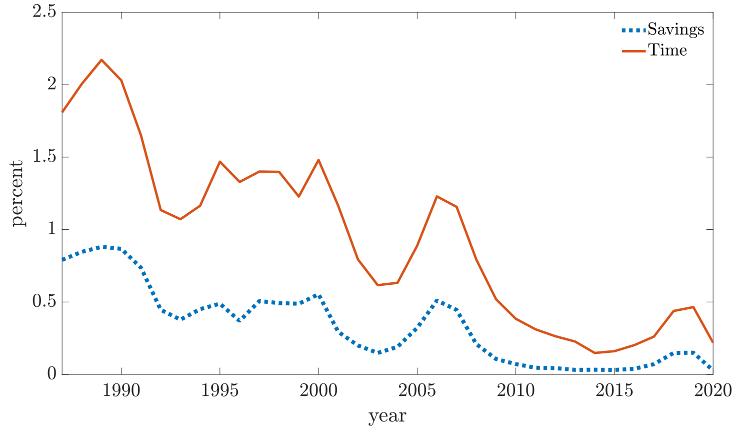
1. *The average interest cost of savings deposits is lower than time deposits.*
2. *The flow of savings deposits is more volatile than time deposits.*
3. *The variance of flows decreases in bank size.*
4. *The share of savings deposits in the balance sheet increases in bank size.*

2.1 Deposit products: trade-off between price and stability

In essence, the key distinctions between savings deposits and time deposits arise from their contract structures. Savings deposits are non-maturing and typically allow unrestricted withdrawals. Conversely, time deposits have a fixed maturity specified in the contract, and early withdrawals incur penalties. While various factors differentiate these deposit types, it remains essential for a profitable banking operation to secure favorable borrowing costs for profitability and ensure a steady funding stream as assets tend to feature longer terms. As a result, our focus centers on deposit rates and deposit flows.

First, Savings deposit pays a lower interest cost than time deposits. The effective (ex-post) interest rate for both deposits is calculated by total interest expense divided by the total balance for each type of deposit. Figure 2 shows the asset-weighted average deposit rates of each deposit type over sample periods. The effective interest rates for time deposits are always higher than savings deposits, while the spread fluctuates over time.

Second, the degree of free withdrawal and the presence of maturity can influence deposit flows. For instance, a depositor needing quick access to funds would withdraw from savings deposits to avoid early withdrawal penalties associated with time deposits, which add an outflow of savings deposits for a bank. Consequently, savings deposits exhibit greater volatility in both inflows and outflows compared to time deposits. The median quarterly



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1987 to 2020. Every aggregation is asset-weighted. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits.

Figure 2: Deposit rates by product types

outflow of savings deposits is 3.5%, which is higher than the 2.7% outflow for time deposits. Additionally, savings deposits have an inflow rate of 5%, surpassing the 3.6% inflow rate for time deposits.[2].

2.2 Deposit mix over size distribution

Table 1 shows the median inflows and outflows of deposits for bank size classes. We categorize the banks into three groups: $\{large,\ medium,\ small\}$. Banks with consolidated assets in the top 0.1% in each period fall into large banks. Between top 0.1% and top 10%, there are medium-sized banks. And the bottom 90% consists of small banks [3]. Across all categories

[2]The growth rates are used as a proxy for the flow of deposits since the direct observation of the flow of deposits is not available. Jermann and Xiang (2023) also discusses the lack of data for the flow of deposits. The quarter-by-quarter growth rate of deposit stock is not a perfect measure for flows but it still enables us to compare the relative volatility of two deposit products in the bank's balance sheet.

[3]For example, in the first quarter of 2021, JPMorgan Chase Bank, Bank of America, Wells Fargo Bank, Citibank, and U.S. bank fall into *large* group. The threshold asset size for the top 10% is \$1,923 million so we have fewer banks than the Fed reports as large commercial banks that have consolidated assets of \$300 million or more. The size classes of banks are somewhat loosely defined but it provides a summarizing picture of asset market shares. As of the first quarter of 2021 (last period of the sample), *large* banks account for 48% of total assets in the banking sector and *medium* banks take 45% and then *small* banks explains the rest, 7%. Among 4,215 banks, only five banks are *large*, and the number of *medium* and *small* is 417 and 3,793, respectively.

		Large	Medium	Small
Outflows	Savings (freq.)	0.88	1.36	1.73
		(0.50)	(0.65)	(0.72)
	Time	0.50	0.98	1.05
		(0.50)	(0.71)	(0.73)
	Total	1.14	1.74	2.11
		(0.50)	(0.66)	(0.72)
Inflows	Savings (freq.)	1.22	1.88	2.21
		(0.50)	(0.63)	(0.68)
	Time	0.52	1.28	1.39
		(0.50)	(0.74)	(0.75)
	Total	1.25	2.34	2.75
		(0.01)	(0.69)	(0.73)

Note: "freq." in parenthesis represents the proportion of outflows or inflows that exceed the median outflow or inflow of large banks within each size group. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits. *Large* is Top 0.1% banks in asset size. *Medium* is Top 10% excluding Top 0.1% banks. *Small* contains banks in bottom 90% of asset percentile.

Table 1: Flows of bank deposits

of deposits, those held in large banking institutions exhibit a higher degree of stability compared to other bank sizes. By evaluating the median flow values specific to large banks, we determine the proportion of instances where the magnitude of flows is greater than or equal to this median benchmark for large banks. It has been observed that this ratio inversely correlates with the size of the bank, diminishing as the bank size increases. Although there are statutory similarities in deposit contracts across different sizes of banks, the flow of deposits behaves differently.

Due to the inherent non-maturity characteristic of savings deposits, banks face constraints in repurchasing these deposits. Consequently, Table 1 suggests that smaller banks may be more vulnerable to substantial withdrawals of savings deposits [4]. Conversely, larger banks benefit from their size, with the Law of Large Numbers contributing to the stability of their deposits. Savings deposits, which are typically susceptible to runs, are actually more secure

[4]Deposit outflows typically occur due to two factors: (i) depositor-initiated withdrawals and (ii) strategic balance sheet reductions by the bank itself. In a later chapter, we discuss the identification of withdrawal from outflow of deposits.

in larger institutions. This study does not delve into the underlying reasons for the reduced outflow rates in large banks. However, one plausible explanation is that large U.S. banks have achieved growth by extending their branch networks across multiple states, rather than concentrating within limited areas, leading to a more diversified deposit base compared to smaller, local banks. This observation aligns with the findings from the exogenous deposit capacity process estimated by Corbae and D'Erasmo (2021).

Heterogeneity in deposit flows across different asset classes implies that an additional unit of deposits can have a disparate impact on the overall deposit flows of large versus small banks. Then, it can be the case that different sizes of banks would exhibit different optimal levels of deposit mix. Indeed, it is observed that the share of savings deposits in the balance sheet rises in bank size. In Figure 3, there is a (both time- and within-size bin) average pattern of the share of savings deposits in individual bank's balance sheets across different size classes. Savings deposit is the major type of deposit for very large banks in the top 0.1%. For them, time deposit takes up slightly less than 20 percent of their balance sheet. In contrast, within smaller banks—specifically those in the lower 90th percentile—time deposits constitute 2/5 of the balance sheet's deposits. This represents a proportion approximately double that of larger banking institutions.

The composition of deposits is crucial for a bank's operations for two primary reasons: profitability and funding stability. As indicated in Figure 3, smaller banks tend to incur higher interest expenses due to the predominance of time deposits, which carry higher interest rates as shown in Figure 2[5]. This reliance on time deposits can erode profitability unless offset by substantial returns on assets. Furthermore, a bank's funding capacity, particularly for long-term loans, is influenced by its deposit mix, as detailed in Table 1. Variations in the stability of deposit flows can, therefore, have a significant impact on the bank's lending abilities.

[5] We confirm this relationship in scatter plots and regression analysis. Details are in Appendix C.

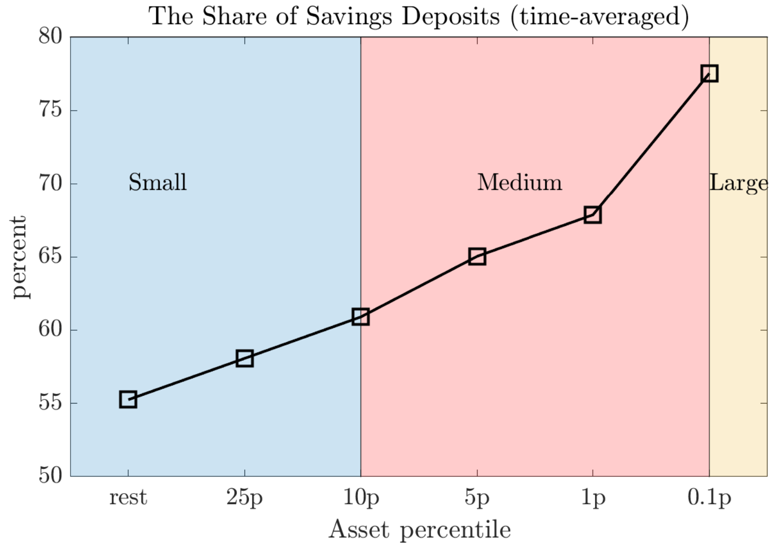


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Figure 3: Share of savings deposit in the balance sheet across bank size distribution

3 Model Economy

We build an equilibrium model of the banking industry with heterogeneous banks to explain the empirical patterns documented earlier and to study their implications on the aggregate economy. Banking industry equilibrium is modeled applying Hopenhayn (1992) to the banking sector of the economy. Therefore, an individual bank's lifecycle is embedded in the model and the banking industry equilibrium features a distribution of heterogeneous banks with endogenously determined bank capital and size of assets and deposits.

The model economy is populated by a representative household, a continuum of heterogeneous banks, and government. Banks engage in liquidity transformation, utilizing their net worth, equity, and two types of deposits to finance loans and securities. Banks have market power over their deposits as in Drechsler et al. (2021) and the markets for loans and securities are perfectly competitive. A representative household valuing both consumption and liquidity chooses consumption and saving with an endowment. Bank deposits deliver bank-specific liquidity services and household allocates deposits across bank distribution given the deposit contracts offered by banks. Households own banks. The government supplies securities as a form of government bonds which carry a risk-free interest rate and are funded through a lump-sum tax on the households.

3.1 Representative household

There is a continuum of identical (equivalently, representative) infinitely-lived households in the economy. Given an endowment (w), stock of deposits ($D_S + D_T$), profit from banks (Π), and lump-sum tax (τ), households consume (C) and make bank deposits to save. Households can save to bank deposits by allocating deposits across a continuum of banks, $j \in (0, 1)$. Since the bank deposit market is assumed to be monopolistically competitive, households are offered a bank-specific deposit price for both savings and time deposits, $\{q_{S,j}, q_{T,j}\}$. Then, the total amount of savings to bank deposit is $\int \left\{q_{S,j}d'_{S,j} + q_{T,j}d'_{T,j}\right\} \mathrm{d}j$. Therefore, the budget constraint of the household at the beginning of the period is given by

$$C + \int \left\{q_{S,j}d'_{S,j} + q_{T,j}d'_{T,j}\right\} \mathrm{d}j \leq w + D_S + D_T + \Pi - \tau$$

where consumption good is a numeraire and $D_S = \int d_{S,j}\mathrm{d}j$ and $D_T = \int d_{T,j}\mathrm{d}j$ are total amount of deposits pre-determined from previous period.

On top of the consumption over the lifetime, households value aggregate liquidity from the bank deposits. Liquidity service and consumption forms a CES composite, Z that enters into the isoelastic utility function, $U(Z)$,

$$Z = \left(\lambda_C^{\frac{1}{\eta}} C^{\frac{\eta-1}{\eta}} + (1 - \lambda_C)^{\frac{1}{\eta}} L^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

where η is the elasticity of substitution between consumption and liquidity service. λ implies the share of consumption goods in the CES composite. Aggregate liquidity comes from the aggregation of bank-specific liquidities (L_j) which we assume to be imperfectly substitutable.

Then, the aggregate liquidity is

$$L = \left(\int \delta_j^{\frac{1}{\nu}} L_j^{\frac{\nu-1}{\nu}} \mathrm{d}j \right)^{\frac{\nu}{\nu-1}}$$

where ν is the elasticity of substitution between liquidity services of banks. δ_j plays as a weight for bank-specific liquidity in the CES aggregator. This reflects the deposit base or liquidity capacity of a bank and somewhat summarizes the geographical availability of bank branches and/or depositors' inertia created by the relationship with bankers or search costs. In the bank's problem, δ_j is an idiosyncratic state that indirectly affects the scale of the operation. Within a bank, households make a portfolio choice between savings deposits and time deposits given discount prices of deposits, $\{q_{S,j}, q_{T,j}\}$, set by a bank. Both as a form of CES composite deliver a bank-specific liquidity service, L_j.

$$L_j = \left(\lambda_S^{\frac{1}{\epsilon}} (d'_{S,j})^{\frac{\epsilon-1}{\epsilon}} + (1-\lambda_S)^{\frac{1}{\epsilon}} (d'_{T,j})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

where ϵ is the elasticity of substitution between saving and time deposits and λ_S reflects the share of savings deposit in the liquidity for bank j.

Denote μ as the distribution of banks and $\mathcal{Q} \equiv \{q_{S,j}, q_{T,j}\}_{j=0}^{1}$ as a set of deposit prices from banks. The value of a representative household is given by

$$V^H(D_S, D_T, \mu, \mathcal{Q}) = \max_{Z,C,L,\{L_j\},\{d'_{S,j}\},\{d'_{T,j}\}} U(Z) + \beta \mathbb{E}\left[V^H(D'_S, D'_T, \mu', \mathcal{Q}') \right] \quad \text{s.t.} \tag{1}$$

$$C + \int \left\{ q_{S,j} d'_{S,j} + q_{T,j} d'_{T,j} \right\} \mathrm{d}j \le w + D_S + D_T + \Pi - \tau \tag{2}$$

$$Z = \left(\lambda_C^{\frac{1}{\eta}} C^{\frac{\eta-1}{\eta}} + (1-\lambda_C)^{\frac{1}{\eta}} L^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{3}$$

$$L = \left(\int \delta_j^{\frac{1}{\nu}} L_j^{\frac{\nu-1}{\nu}} \mathrm{d}j \right)^{\frac{\nu}{\nu-1}} \tag{4}$$

$$L_j = \left(\lambda_S^{\frac{1}{\epsilon}} (d'_{S,j})^{\frac{\epsilon-1}{\epsilon}} + (1-\lambda_S)^{\frac{1}{\epsilon}} (d'_{T,j})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{5}$$

3.2 Heterogeneous banks

At the beginning of each period, a bank starts the operation with loans (ℓ), government bonds (a), outstanding savings deposits (d_S), and time deposits (d_T) in the balance sheet. From the balance sheet identity, the bank's net worth at the beginning of the period (n) is

$$n = \ell + a - d_S - d_T.$$

To earn profit, banks invest in one-period assets. Both loans ($\tilde{\ell}'$) and government bonds ($\tilde{a}'$) which return one at the beginning of the next period can be purchased at the discount price q_ℓ and q_a, respectively in the perfectly competitive markets. Banks use net worth and source funds through deposits ($\tilde{d}'_S + \tilde{d}'_T$) and equity issuance ($e > 0$). Equity issuance incurs a non-pecuniary cost and it is valued in $\psi(e)$. The cost of equity issuance induces a bank to issue deposits and helps to produce a non-trivial capital structure for the bank. Bank deposits are non-defaultable one-period debts and fully insured in the case of bank failure. Each bank has market power over deposits so deposit contracts offered by banks feature discounted prices and the quantity of deposits. The contract of savings deposit is $\{q_S, \tilde{d}'_S\}$ and contract of time deposit is $\{q_T, \tilde{d}'_T\}$. Bank pays a dividend ($e < 0$) and it is valued in $\psi(e)$. Then, the flow budget at the beginning of the period is

$$q_\ell \tilde{\ell}' + q_a \tilde{a}' - e \leq n + q_S \tilde{d}'_S + q_T \tilde{d}'_T \tag{6}$$

In the deposit market, each bank has market power over its deposits. It enables a bank to set interest rates for deposits given the household's demand functions. Liquidity capacity (δ) is an idiosyncratic state of a bank and it scales the household's demand functions, in turn, determines the scale of the bank's balance sheet. The sizes of assets and deposits are endogenously determined in the model. Yet, the size type of the bank in terms of liquidity capacity helps us to match the model to data. Later, we show that the classification of bank size classes is evident with estimated withdrawal shocks. AR(1) term creates uncertainty and it helps to generate heterogeneity within each group of banks with the same types.

The main difference between savings and time deposits from the perspective of the bank is the degree of exposure to the withdrawal risk. Since both deposits are one-period debt, it is assumed that there is a withdrawal shock in the middle of a period for savings deposits. In contrast, a time deposit is free from the withdrawal risk. So, a bank is obligated to service the withdrawal of savings deposits in the middle of each period before assets mature. The withdrawal risk in savings deposits makes a bank face a liquidity risk. We model the withdrawal as an idiosyncratic shock, $\theta \in (0,1)$ that is i.i.d. with a cumulative density function, $F(\theta|\delta)$ where $\partial \mathrm{E}(\theta|\delta)/\partial\delta < 0$ and $\partial \mathrm{Var}(\theta|\delta)/\partial\delta < 0$. As we make the distribution of withdrawal shock depend on the liquidity capacity, this captures the lower outflows in saving deposits for large banks relative to other banks as documented in data.

Withdrawal of saving deposits, $\theta\tilde{d}'_S$, is serviced by selling the assets at the market prices. We assume that loans are illiquid and government bonds are liquid in the asset market in the sense that selling loans accompanies a discount and the value of loans is partially recovered, $\omega q_\ell \tilde{\ell}'$ where $1-\omega \in (0,1)$ is a fraction of loss with a discount. Whereas, the value of government bonds (securities) is recovered at the current market price without any discount, $q_a\tilde{a}'$. This gives an incentive for a bank to hold securities in normal times although their returns are lower than loans.

Meeting the withdrawal request is assumed to follow a pecking-order decision as in Supera (2021). Securities come first to serve the withdrawal and if the amount of withdrawal exceeds the current market value of securities in the balance sheet, the bank starts selling loans with a discount. The model analysis in the next section shows that the pecking-order decision is optimal for a particular order of asset prices. Let ι denote the policy function of the sale of loans ($\iota = 1$, otherwise $\iota = 0$). The pecking-order decision can be expressed as

$$\begin{cases} \iota = 0, & \text{if } \theta\tilde{d}'_S \leq q_a\tilde{a}' \\ \iota = 1, & \text{if } \theta\tilde{d}'_S > q_a\tilde{a}' \end{cases}$$

Following the realization of the withdrawal shock, the bank's assets are re-balanced. Then, the bank's balance sheet for the next period is determined with re-balanced assets and remaining savings deposits as $\{\ell', a', d'_S, d'_T\}$. The balance sheet next period implies bank's net worth for the next period satisfies

$$n' = \ell' + a' - d'_S - d'_T$$

Re-balanced assets and remaining liabilities differ by the size of the withdrawal shock and deposit mix. Based on the pecking-order decision, the change in the balance sheet is summarized as follows,

i) $\iota = 0$ when $\theta \tilde{d_S}' \leq q_a \tilde{a}'$,

$$\{\ell', a', d'_S, d'_T\} = \begin{cases} \ell' & = \tilde{\ell}' \\ a' & = \tilde{a}' - \theta \tilde{d_S}'/q_a \\ d'_S & = (1-\theta)\tilde{d_S}' \\ d'_T & = \tilde{d_T}' \end{cases}$$

ii) $\iota = 1$ when $\theta \tilde{d_S}' > q_a \tilde{a}'$,

$$\{\ell', a', d'_S, d'_T\} = \begin{cases} \ell' & = \tilde{\ell}' - (\theta \tilde{d_S}' - q_a \tilde{a}')/(\omega q_\ell) \\ a' & = 0 \\ d'_S & = (1-\theta)\tilde{d_S}' \\ d'_T & = \tilde{d_T}' \end{cases}$$

While the bank operates, the bank is regulated at the time of investment decision is made such that it must hold sufficient bank capital as a fraction of risk-weighted assets. The capital requirement constraint is

$$q_\ell \tilde{\ell}' + q_a \tilde{a}' - q_S \tilde{d}_S' - q_T \tilde{d}_T' \geq \chi \omega_\ell q_\ell \tilde{\ell}'$$

where χ is the capital requirement and ω_ℓ is risk-weight specified by the regulation.

The objective of the bank is to maximize the expected present discounted value of future net dividend payouts with a valuation function, $\psi(e)$. Since the representative household owns the bank, future values are discounted using the same discount factor as the household. Before the beginning of the next period, every incumbent bank faces an exogenous exit shock, $1-\pi \in (0,1)$, which prevents a bank from accumulating a large enough capital to stop issuing the deposits. The exited banks are replaced with the entrants of the same mass with zero net worth.

The value of an incumbent bank is given by

$$V^0(n, \delta; \mu, \mathcal{Q}) = \max_{\tilde{\ell}', \tilde{a}', q_S, q_T, e} \psi(e) + \mathbb{E}_\theta \left[V^1(\tilde{\ell}', \tilde{a}', \tilde{d_S}', \tilde{d_T}', \delta, \theta) \right] \tag{7}$$

subject to

$$q_\ell \tilde{\ell}' + q_a \tilde{a}' - e \leq n + q_S \tilde{d_S}' + q_T \tilde{d_T}'$$

$$q_\ell \tilde{\ell}' + q_a \tilde{a}' - q_S \tilde{d_S}' - q_T \tilde{d_T}' \geq \chi q_\ell \tilde{\ell}'$$

$$\tilde{d_S}' = \tilde{d_S}'(q_S, q_T, \delta; \mu, \mathcal{Q}) \;\;\&\;\; \tilde{d_T}' = \tilde{d_T}'(q_S, q_T, \delta; \mu, \mathcal{Q})$$

and

$$V^1(\tilde{\ell}', \tilde{a}', \tilde{d_S}', \tilde{d_T}', \delta, \theta) = \beta\pi\mathbb{E}V^0(n', \delta'; \mu', \mathcal{Q}') \tag{8}$$

subject to

$$n' = \ell' + a' - d'_S - d'_T$$

$$\ell' = \tilde{\ell}' - \iota(\tilde{a}', \tilde{d}'_S, \theta)(\theta\tilde{d}'_S - q_a\tilde{a}')/(\omega q_\ell)$$

$$a' = (1 - \iota(\tilde{a}', \tilde{d}'_S, \theta))(\tilde{a}' - \theta\tilde{d}'_S/q_a)$$

$$d'_S = (1-\theta)\tilde{d}'_S$$

$$d'_T = \tilde{d}'_T$$

$$\iota(\tilde{a}', \tilde{d}'_S, \theta_T) = \begin{cases} 0\ , & \text{if} \ \ \theta\tilde{d}'_S \leq q_a\tilde{a}' \\ 1\ , & \text{if} \ \ \theta\tilde{d}'_S > q_a\tilde{a}' \end{cases}$$

where the value function, V^0, is defined over the individual state $\{n, \delta\}$. μ is the distribution of banks and $\mathcal{Q} = \{q_{S,j}, q_{T,j}\}_{j=0}^{1}$ is a set of deposit prices offered by banks in the economy. $\mathcal{Q}$ is now the aggregate state variable because each bank has a market power in the deposit market and competes with each other to attract deposits from the household.

3.3 Distribution of banks

The distribution of heterogeneous banks is defined over $m \equiv \{n, \delta\}$. The evolution of the distribution is summarized as a functional operator $T\mu$ and it is given by

$$\begin{aligned}\mu'(\hat{n}', \delta') &= T\mu \\ &= \pi\Pi_\delta(\delta'|\delta)\Bigg[\int \mathbf{1}\Bigg\{g_\ell(m) = \tilde{\ell}', g_a(m) = \tilde{a}', \tilde{d}'_S = \tilde{d}'_S(g_{q_S}(m), g_{q_T}(m)), \\ &\tilde{d}'_T = \tilde{d}'_T(g_{q_S}(m), g_{q_T}(m))\Bigg\} \times \Pi_\theta\mathbf{1}\left\{n' = \hat{n}'|\theta, g_\iota\right\}\mu(m)\mathrm{d}m\Bigg] \\ &+ \mathbf{1}\left\{\hat{n}' = 0 \ \& \ \delta' = \min(\delta)\right\}(1-\pi)\mu \end{aligned} \tag{9}$$

for all $\{\hat{n}', \delta'\}$. Π_δ and Π_θ are the density of δ and θ, respectively. $\mathbb{1}\{\cdot\}$ is an indicator function that produces 1 when the statement is true. $\{g_\ell, g_a, g_{q_S}, g_{q_T}\}$ are the optimal policy functions for loans, securities, savings deposit price, and time deposit price. The stationary distribution at the steady state satisfies, $\mu = T\mu$.

3.4 Government

The government in this economy levies lump-sum tax (τ, transfer if negative) from the representative household to elastically supply the government bonds to banks. Define $A_B \equiv \int a'(n, \delta') d\mu(n, \delta')$ as the aggregate demand for government bonds. The equation for the balanced budget is $q'_a A'_B + \tau = A_B$. The required tax to meet the aggregate demand is given by

$$\begin{aligned}
\tau(A_B, A'_B, q'_a) &= A_B - q'_a A'_B \\
&= \int a'(n, \delta') d\mu(n, \delta') - q'_a \int a''(n, \delta') d\mu'(n, \delta')
\end{aligned}$$

In the stationary equilibrium, $A'_B = A_B$ and $q_a = q'_a$,

$$\tau(A_B, q_a) = (1 - q_a) A_B \tag{10}$$

3.5 Timing

In any period, the following stages occur.

Stage 1. Given the distribution of the bank μ, exogenous exit shock $(1 - \pi)$ is realized.

- Entrants replace exited banks with net worth, $n_e = 0$.
- Incumbent banks start operating with pre-determined net worth.

Stage 2.

- Banks: Given their net worth and liquidity capacity, banks strategically allocate resources by choosing loans, securities, deposits, and determining dividend/equity issuance, $(\tilde{\ell}', \tilde{a}', \tilde{d}'_S, \tilde{d}'_T, e)$, expecting the withdrawal risk during the interim period.

- Households: given the cash-on-hand ($\equiv w + D_S + D_T + \Pi - \tau$), households choose consumption and make deposits across banks.

Stage 3. Withdrawal shock θ is realized and banks carry out re-balancing assets to meet the withdrawal request.

Stage 4. At the start of the next period, assets and deposits mature thereby net worth is determined.

The same timing assumption of the model is depicted in Figure 4.

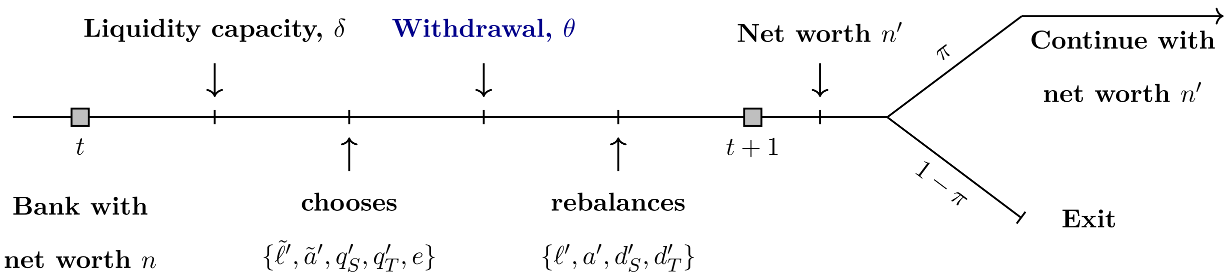


Figure 4: Timing of the bank problem

3.6 Definition of equilibrium

A stationary recursive equilibrium is a set of functions

$$\left\{V^0, V^1, g_\ell, g_a, g_{d_S}, g_{d_T}, g_e, \iota, \mu, L_B, A_B, D'_{S,B}, D'_{T,B}, V^H, \{d'_{S,j}\}, \{d'_{T,j}\}, \tau, C, Z, L, \{L_j\}, w, \Pi\right\}$$

and prices $\{q_\ell, q_a\}$ such that

(i) Given prices $\{q_\ell, q_a\}$, $\{g_\ell, g_a, g_{q_S}, g_{q_T}, g_e, \iota\}$ solves the bank problem, 7 and 8.

(ii) Given deposit rate offers $\mathcal{Q}$, $\left\{\{d'_{S,j}\}, \{d'_{T,j}\}, C, L, \{L_j\}, Z\right\}$ solves the household problem, 1.

(iii) Distribution of banks (μ) evolves following 9 and stationary distribution satisfies $\mu = T\mu$.

(iv) Deposit market clears:

- savings deposits: $D_S = \int \tilde{d}'_S(g_{q_S}(n,\delta), g_{q_T}(n,\delta))\mathrm{d}\mu(n,\delta)$
- time deposits: $D_T = \int \tilde{d}'_T(g_{q_S}(n,\delta), g_{q_T}(n,\delta))\mathrm{d}\mu(n,\delta)$

(v) Government's budget satisfies as in 10 that sustains the aggregate demand for the government bond at price q_a.

(vi) Loan market clears with a perfectly elastic demand at q_ℓ.

4 Model Implications

This section analyzes model implications. First, we see how withdrawal risk in savings deposits is related to bank capital. We discuss the optimality of the pecking-order decision rules. Next, in the household's problem. We show that the liquidity premium on deposits arises endogenously because the household values bank liquidities in the utility function. Finally, we describe the deposit demand function of the household.

4.1 The effect of withdrawal risk on the bank capital

At the beginning of the period, the bank's choices of assets and deposits define the maximal net worth for the next period, $\overline{n}'$,

$$\overline{n}' = \tilde{\ell}' + \tilde{a}' - \tilde{d}'_S - \tilde{d}'_T$$

Note that this is a hypothetical net worth that is attainable in scenarios where withdrawal shocks are absent. After observing the withdrawal shock, the realized net worth for the next period affected by re-balanced assets is

$$n'_0 \equiv n'|_{\iota=0} = \tilde{\ell}' + \tilde{a}' - \tilde{d}'_S - \tilde{d}'_T - \left(1 - \frac{1}{q_a}\right)\theta\tilde{d}'_S$$

$$n'_1 \equiv n'|_{\iota=1} = \tilde{\ell}' + \frac{q_a}{\omega q_\ell}\tilde{a}' - \tilde{d}'_S - \tilde{d}'_T - \left(1 - \frac{1}{\omega q_\ell}\right)\theta\tilde{d}'_S$$

Proposition 1 *The realized net worth after the withdrawal of saving deposits is smaller than the maximal net worth.*

Proof. see Appendix A. □

The loss in net worth because of withdrawal shock includes foregone interest income from the assets as they are sold earlier than their maturity. The loss can be large when the illiquid loans are used or the risk-free rate is high. Thus, in the low interest rate economy, the bank would not face much net worth loss. However, the loss can be great and the liquidity risk is elevated as the risk-free rate rises. When a bank faces a large withdrawal in savings deposits, of course, raises the loss more.

4.1.1 Is it optimal to follow the pecking-order decision rule?

The bank in the model economy is assumed to follow the pecking-order decision rule to satisfy the withdrawal request on savings deposits: government bonds come first to meet the withdrawal and if the current market value of government bonds is insufficient, loans are used with a partial recovery. This pecking-order decision rule implies a threshold policy for selling the loans. Given the choices $\{\tilde{a}', \tilde{d}'_S\}$ and the realization of withdrawal shock, θ, banks start selling loans only if $\theta\tilde{d}_S > q_a\tilde{a}'$. Therefore, we can define a threshold value of withdrawal shock as $\overline{\theta}(\tilde{a}', \tilde{d}'_S; q_a) \equiv \frac{q_a\tilde{a}'}{\tilde{d}'_S}$. The pecking-order decision rule can be described as

$$\iota(\tilde{a}', \tilde{d}_S, \theta) = \begin{cases} 0, & \text{if } \theta \leq \overline{\theta}(\tilde{a}', \tilde{d}'_S; q_a) \\ 1, & \text{if } \theta > \overline{\theta}(\tilde{a}', \tilde{d}'_S; q_a) \end{cases}$$

In the interim period after observing the withdrawal shock, paying out the withdrawal request by liquid government bonds reduces the net worth smaller than selling out illiquid loans to preserve the bank net worth next period if $\omega q_\ell < q_a$. In other words, selling the liquid assets first following the pecking-order rule can be optimal because both assets are one-period and capital requirement is already considered at the moment of choosing $\{\tilde{\ell}', \tilde{a}', \tilde{d}'_S, \tilde{d}'_T\}$. Therefore, the optimal policy of repaying the withdrawal of saving deposits is such that maximizes the net worth of the next period since the value function at the beginning of the next period is increasing in net worth [6]. To minimize the loss in the bank capital next period, it is optimal to hold sufficiently large securities so that banks can fully insure the risk of selling loans with discounts so there is a precautionary motive for purchasing liquid assets.

Corollary 1 *Loss in bank capital is greater for a unit increase in withdrawal of savings deposits when loans are used than meeting withdrawal using only securities,* $\left|\frac{\partial n'_0}{\partial \theta \tilde{d}'_S}\right| < \left|\frac{\partial n'_1}{\partial \theta \tilde{d}'_S}\right|$

Proof. see Appendix A □

4.2 Endogenous liquidity premium and bank deposit demand

4.2.1 Liquidity service in utility function

The households value liquidity services in the utility function. Liquidity service consists of CES composite of savings and time deposits. The Euler equations from the household problem are

$$q_{S,j}\Omega = \beta\mathbb{E}[\Omega'] + \frac{\partial U}{\partial d'_{S,j}}$$
$$q_{T,j}\Omega = \beta\mathbb{E}[\Omega'] + \frac{\partial U}{\partial d'_{T,j}}$$

[6] However, the pecking-order decision rule can be suboptimal in a more general environment. Later in this part, we are going to come back and check the optimality of the pecking order decision in the full-blown model (with different timing of capital requirement and/or long-term assets).

where Ω is the Lagrange multiplier for the budget constraint. The second term on the right-hand side of the Euler equations shows an additional value directly delivered to the utility function. Given that $\frac{\partial U}{\partial d'_{S,j}} > 0$ and $\frac{\partial U}{\partial d'_{T,j}} > 0$ and focusing on the stationary economy where $\Omega' = \Omega$, liquidity premium exists and so $q_{S,j} > \beta$ and $q_{T,j} > \beta$. Considering the ratio of two marginal utilities, we can compare the relative size of liquidity premium for each type of deposit.

$$\frac{\frac{\partial U}{\partial d'_{S,j}}}{\frac{\partial U}{\partial d'_{T,j}}} = \frac{\frac{\partial U}{\partial Z}\frac{\partial Z}{\partial L}\frac{\partial L}{\partial L_j}\frac{\partial L_j}{\partial d'_{S,j}}}{\frac{\partial U}{\partial Z}\frac{\partial Z}{\partial L}\frac{\partial L}{\partial L_j}\frac{\partial L_j}{\partial d'_{T,j}}} = \frac{\frac{\partial L_j}{\partial d'_{S,j}}}{\frac{\partial L_j}{\partial d'_{T,j}}} = \frac{\lambda_S^{\frac{1}{\epsilon}}(d'_{S,j})^{-\frac{1}{\epsilon}}}{(1-\lambda_S)^{\frac{1}{\epsilon}}(d'_{T,j})^{-\frac{1}{\epsilon}}}$$

If we have very large λ_S, then $q_{S,j} > q_{T,j}$ is available. If either type of deposit brings a larger marginal effect on liquidity service, then the liquidity premium is higher, which makes the discount price further from the risk-free price, β.

4.2.2 Bank-level deposit demand and allocating over distribution

The solution to the household problem in the stationary economy provides the deposit demand function for bank j as a function of cash-on-hand and discount prices of deposits.

$$d'^*_{S,j}(q_{S,j}, q_{T,j}, L_j) = \lambda_S(q_{S,j} - \beta)^{-\epsilon}\,\overline{q}_j(q_{S,j}, q_{T,j})L_j \tag{11}$$

$$d'^*_{T,j}(q_{S,j}, q_{T,j}, L_j) = (1-\lambda_S)(q_{T,j} - \beta)^{-\epsilon}\,\overline{q}_j(q_{S,j}, q_{T,j})L_j \tag{12}$$

where $\overline{q}_j(q_{S,j}, q_{T,j}) \equiv (\lambda_S(q_{S,j} - \beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j} - \beta)^{1-\epsilon})^{\frac{\epsilon}{1-\epsilon}}$.

Liquidity service from bank j is

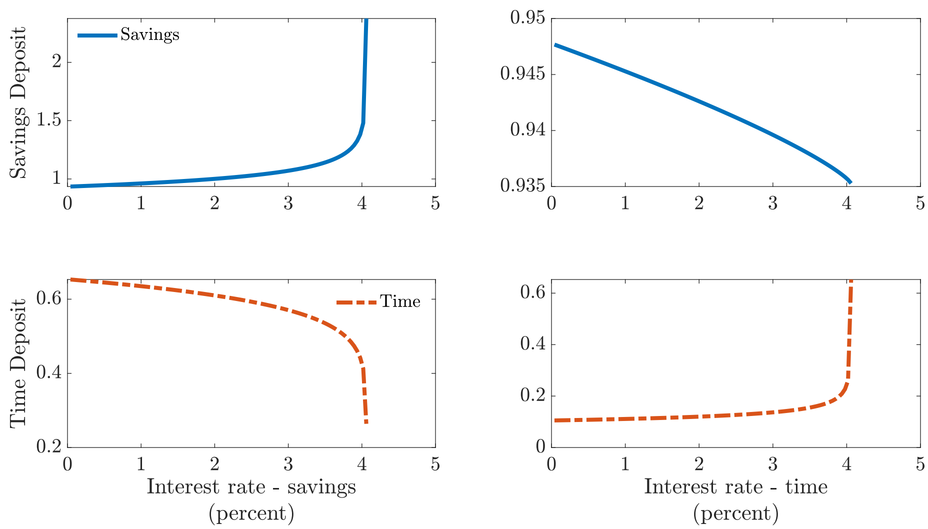
$$L^*_j(P_{L,j}, P_L, L; \delta_j) = \delta_j \left(\frac{P_{L,j}}{P_L}\right)^{-\nu} L \tag{13}$$

where $P_{L,j}$ is the price index for liquidity service from bank j and P_L is the price for aggregate liquidity as follows,

$$P_{L,j} = \left[\lambda_S(q_{S,j} - \beta)^{-\epsilon} q_{S,j} + (1 - \lambda_T)(q_{T,j} - \beta)^{-\epsilon} q_{T,j}\right] \overline{q}_j(q_{S,j}, q_{T,j}) \tag{14}$$

$$P_L = \left[\int \delta_j P_{L,j}^{1-\nu} \mathrm{d}j\right]^{\frac{1}{1-\nu}} \tag{15}$$

Note that bank-specific liquidity decreases in relative price, $P_{L,j}/P_L$ because banks compete for deposits and depositors optimally allocate bank-specific liquidity service based on its relative price. The bank distribution and a set of deposit prices determined by banks affect how much households allocate liquidity services across banks. Therefore, $\mathcal{Q} = \{q_{S,j}, q_{T,j}\}$ becomes a relevant aggregate state variable. Liquidity capacity, δ_j, scales L_j, and then each deposit demand level. This helps a bank to become large without charging a higher deposit rate than other banks [7].



Notes: For the expositional purpose, the Left panel fixes the interest rate for time deposit at 4% and the right panel sets the saving deposit rate at 0.04%. This figure uses $\{\lambda_S, \beta, \epsilon, \nu, \delta, L, P_L\} = \{0.9, 1/1.01, 0.2, 0.1, 1.0, 0.65, 1.05\}$.

Figure 5: Household deposit demand function

[7] Going back to data, we do not observe large banks in the U.S. attract depositors by offering high interest rates for deposits. Rather they put markdown on deposits with a stronger deposit market power as documented in Drechsler et al. (2021). The liquidity capacity term, hence, models attributes from large banks that attract customers and prevent turnover. For example, relationship, depositor's inertia (habit), expansions of branches, etc.

Figure 5 depicts a household's deposit demand function for an individual bank. Upper-left and bottom-right panel shows that deposit demand rises in their own interest rates. Since savings and time deposits are imperfectly substitutable, time deposit demand falls as the savings deposit rate increases as in the bottom-left panel. Likewise, savings deposit demand decreases in time deposit rate which can be found in the upper-right panel.

5 Mapping the Model to the Data

To quantitatively study the role of bank deposit structure in the aggregate economy, specific functional forms are assumed and we assign parameter values. Later, a set of parameters is going to be internally estimated for the calibration. For now, all parameters are externally set.

5.1 Withdrawal shock inferred from data

5.1.1 Model-based approach

Withdrawal of deposits is not directly observed in the data and so we recover and estimate the withdrawal shock from the bank's deposit flows. Before we use our model to recover the withdrawal shock, we need an assumption about the timing of observation of data because the model economy has interim period withdrawal shock and then the bank's balance sheets between the beginning and the end of the period are different. We assume that data reflects the end-of-period balance sheet of the bank, which enables us to get the information about realized withdrawal shock.

At the end of the period, for any withdrawal shock, $\theta \in \Theta$, the remaining balances for securities and savings deposits are

$$a' = \tilde{a}' - \frac{\theta \tilde{d}'_S}{q_a}$$

$$d'_S = (1 - \theta)\tilde{d}'_S$$

From the model, we know that securities are demanded to insure against withdrawal shock and the realization of withdrawal shock changes the balances of securities and savings deposits. Thus, we consider their ratio to retrieve the realized withdrawal shock, a'/d'_S. Then,

$$
\begin{aligned}
\frac{a'}{d'_S} &= \frac{\tilde{a}' - \frac{\theta \tilde{d}'_S}{q_a}}{(1-\theta)\tilde{d}'_S} \\
\Rightarrow \frac{q_a a'}{d'_S} &= \frac{q_a \tilde{a}' - \theta \tilde{d}'_S}{(1-\theta)\tilde{d}'_S} \\
\Rightarrow (1-\theta)\frac{q_a a'}{d'_S} &= \frac{q_a \tilde{a}'}{\tilde{d}'_S} - \theta \qquad (16)
\end{aligned}
$$

Let's call the ratio, $q_a a'/d'_S$, as operational liquidity ratio (OLR). Then the equation 16 relates operational liquidity ratios between the beginning and the end of the period to the realized withdrawal shock. Securities in the model are demanded to prevent the sale of loans provided that the recovery value of loans is strictly smaller than the market value of securities and the bank's value function is weakly increasing in bank capital. The optimal securities demand implies that

$$
\begin{aligned}
q_a \tilde{a}' &\geq \theta \tilde{d}'_S \\
\Rightarrow q_a \tilde{a}' &= \overline{\theta} \tilde{d}'_S \qquad (17)
\end{aligned}
$$

where $\overline{\theta} \equiv \max \Theta$. Combining 16 and 17 yields

$$
\begin{aligned}
(1-\theta)\frac{q_a a'}{d'_S} + \theta &= \overline{\theta} \\
\Rightarrow \frac{\overline{\theta} - \theta}{1-\theta} &= \frac{q_a a'}{d'_S} \qquad (18)
\end{aligned}
$$

Equation 18 can be used to recover θ given $\overline{\theta}$ and the operational liquidity ratio.

5.1.2 Withdrawal shock and kernel density estimation

For implementing the identification method using 18, let's re-express it again for observation level (bank(i), time(t)) subscript.

$$\frac{\overline{\theta}_{it} - \theta_{it}}{1 - \theta_{it}} = \frac{q_{a,t}a_{it}}{d_{S,it}} \tag{19}$$

Based on 18, θ_{it} is identified into two steps. First, the upper bound of withdrawal shock is the largest value of the operational liquidity ratio. As a limit case, if $\theta_{it} = 0$, $\dfrac{q_{a,t}a_{it}}{d_{S,it}} = \overline{\theta}_{it}$. Note that the operational liquidity ratio is decreasing in θ_{it}. Then, we can think of the largest operational liquidity ratio as the case where a bank chooses large enough securities to cover the worst withdrawal shock but the actual realization is zero. Suppose that banks in the same period have the same upper bound, $\overline{\theta}_{it} = \overline{\theta}_t$. Then $\overline{\theta}_t = \max\left\{\dfrac{q_{a,t}a_{it}}{d_{S,it}}\right\}$. Next, we can directly use 19 to retrieve $\{\theta_{it}\}$ [8].

$$\theta_{it} = \frac{\overline{\theta}_t - q_{a,t}a_{it}/d_{S,it}}{1 - q_{a,t}a_{it}/d_{S,it}}$$

Finally, given the full set of withdrawal shock $\{\theta_{it}\}$, the cumulative density function, $F(\theta)$, is estimated using the kernel density estimation. In practice, we argue that the withdrawal shock distribution differs across the size classes. As in the previous part of the paper, we categorize the banks into three groups: large (top 0.1%), medium (top 10% - top 0.1%), and small (bottom 90%), $g = \{\ell, m, s\}$. Then, we have $\overline{\theta}_{gt}$ and the rest of the procedure is the same.

In reality, banks can hold liquid assets for other reasons. So, it may be unnatural to assume $q_a a'/d'_S = 0$ in any period. In other words, It would be more appropriate to think that it is not very likely that a bank faces $\theta = \overline{\theta}$. From a distribution perspective, $\overline{\theta}$ is the right-tail. So, we do not expect a meaningful non-zero measure there. Hence, we exclude the observation of $\theta = \overline{\theta}$ after computing the upper bound in the first step. Kernel density

[8] $q_{a,t} = \frac{1}{1+r_{a,t}}$ holds and we use Federal Funds Rates with quarterly frequency for $r_{a,t}$.

estimation is used to figure out the distribution of withdrawal shock, $F(\theta_g)$ [9]. The estimated kernel density shows a very small right-tail probability. We compare kernel density estimates with uniform distribution in Figure 6.

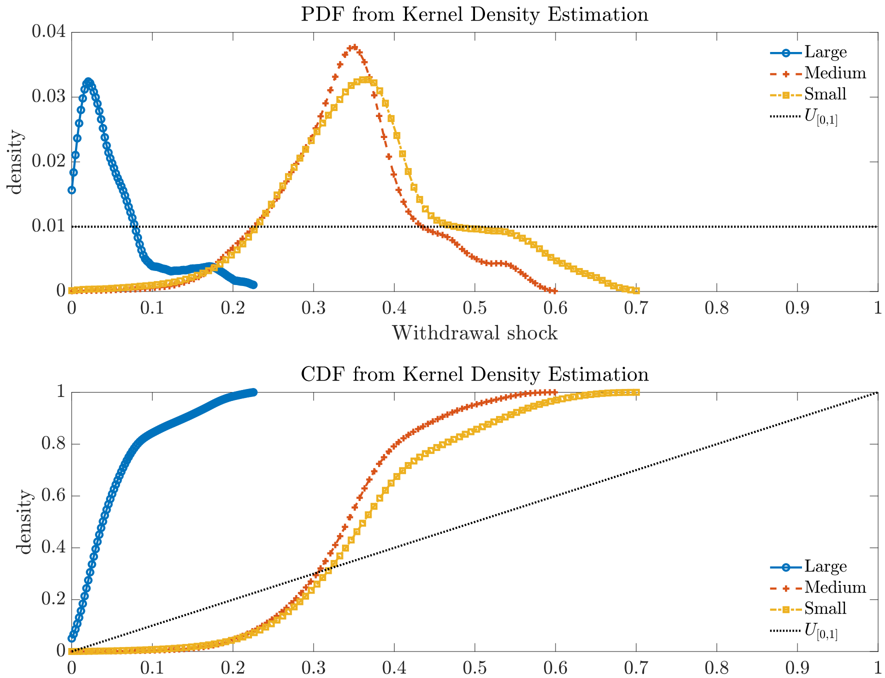


Figure 6: Kernel Density Estimates: PDF and CDF

5.2 Functional forms

We assume the household's utility function is isoelastic and it is $U(Z) = \frac{Z^{1-\gamma}}{1-\gamma}$. Bank values dividend payout and equity issuance using $\psi(e)$. Dividend is valued linearly and equity issuance brings non-pecuniary cost. We assume the following functional form:

$$\psi(e) = \begin{cases} -e & , \text{ if } e \leq 0 \\ -e - \overline{\psi}e & , \text{ if } e > 0 \end{cases}$$

[9] We use the Epanechnikov kernel function which is optimal in a mean squared error sense among many kernel functions. The bandwidth is a free parameter. The optimal width is selected to minimize the mean integrated squared error in normal densities. The kernel density estimates are evaluated at evenly-spaced grids with the minimum and the maximum value of $\{\theta_{it}\}$ as the lower and upper bounds for each asset group, g.

As documented in Dempsey and Faria-e Castro (2022), this specification is widely used in dynamic corporate finance studies and helps to prevent a bank from issuing equity when the capital requirement is slack.

Liquidity capacity follows

$$\delta = \delta_i \delta_p$$
$$\delta'_p = \mu_\delta + \rho_\delta \delta_p + \epsilon_\delta, \quad \epsilon_\delta \sim N(0, \sigma^2_{\epsilon,i})$$

where δ_i is the permanent component of liquidity capacity and δ_p is a persistent component following AR(1) process. The shock has a type-specific term, $\sigma_{\epsilon,i}$, which helps to explain the size variations within each group of banks. Permanent component of liquidity capacity has three values, $\delta_i = \{\delta_\ell, \delta_m, \delta_s\}$ with subscript $\{\ell, m, s\}$ denotes large, medium, and small-sized banks. Permanent type of liquidity capacity captures the difference in near-permanent scale of each bank so that we can separate banks with economy-wide branches, regional banks, and community banks, for example.

5.3 Parameterization

5.3.1 External parameters

The model period is a quarter. A household's risk-aversion coefficient γ is not used for the solution of the stationary equilibrium since there is no uncertainty in the household's problem. Household's time discount factor, β is set to match with 4% of the annual risk-free interest rate. Then, we have $\beta = q_a = 1/(1 + r_f)$. The capital requirement χ is 8% which is in line with current regulation for large bank holding companies in the U.S.. Bank exit probability is equal to the average quarterly bank failure rate, $1 - \pi = 0.72\%$. The loan price is set to target the annual loan rate is equal to 5%. The endowment for the household, w, is normalized to 1. For the AR(1) term of bank liquidity capacity (δ_p), the coefficient targets the average bank deposit retention rate which is 84%. Deposit retention is considered as household weighing aggregate liquidity with the same δ_p. Thus, the average of diagonal element of transition matrix for δ_p is set to be 0.84. For the permanent type of bank, we make it δ_i have a mean equals to one. To this end, $p_\ell \delta_\ell + p_m \delta_m + p_s \delta_s = 1$ must hold. The fraction of each type of banks is based on the definition of large, medium, and small banks we used in data, $\{0.001, 0.099, 0.9\}$.

5.3.2 Internally determined parameters

There are parameters that are jointly determined by targeting the moments. All parameter values used in the model are summarized in Table 2.

	Description	Value	Target	Model	Data
External Parameters					
χ	Capital requirement	0.08	U.S. bank regulation		
ω	Recovery rate for loans	0.8			
π	Survival probability	0.9928	Failure rate, 0.72%		
q_ℓ	Loan price	0.9756	Annual loan rate, 5%		
q_a	Gov't bond price	0.9901	Annual risk-free rate, 4%		
w	Endowment	1.0	Normalization		
ρ_δ	AR(1) coefficient for δ_p	0.9127	Bank deposit retention rate, 84%		
p_l	fraction $\rho_i = \ell$ of banks	0.001	By definition		
p_m	fraction $\rho_i = m$ of banks	0.099	"		
p_s	fraction $\rho_i = s$ of banks	0.9	"		
Internal Parameters					
$\overline{\psi}$	Equity issuance cost	0.5	Bank leverage	0.926	0.877
λ_C	Share of consumption in Z	0.25	Average NIMs	0.042	0.018
λ_S	Share of savings deposits in L_j	0.9	Equity issuance / net worth	0.000	0.011
η	Elasticity of subst. of C and L	0.5	Net dividend/ net worth	0.139	0.058
ν	Elasticity of subst. of L_j	1.1	Consumption-deposit ratio	0.326	0.320
ϵ	Elasticity of subst. of $d_{S,j}$ and $d_{T,j}$	4.5	Spread between d_S and d_T	0.011	0.024
$\sigma_{\delta,g}$	SD of liquidity cap. ($g = \{s, m, \ell\}$)	{0.07,0.1,0.05}	SD of deposits	0.267	2.375
δ_g	Type of liquidity cap.	{0.941, 1.523, 1.843}	SD of deposit rates	0.0003	0.167
			Avg. share of savings deposits	0.558	0.667
			SD of deposits within $g = \ell$	0.211	0.621
			SD of deposits within $g = m$	0.365	1.347
			SD of deposits within $g = s$	0.138	0.738
			Size ratio, ℓ *to* m	1.383	1.235
			Size ratio, m *to* s	1.763	1.546

Table 2: Parameters and Targeted Moments

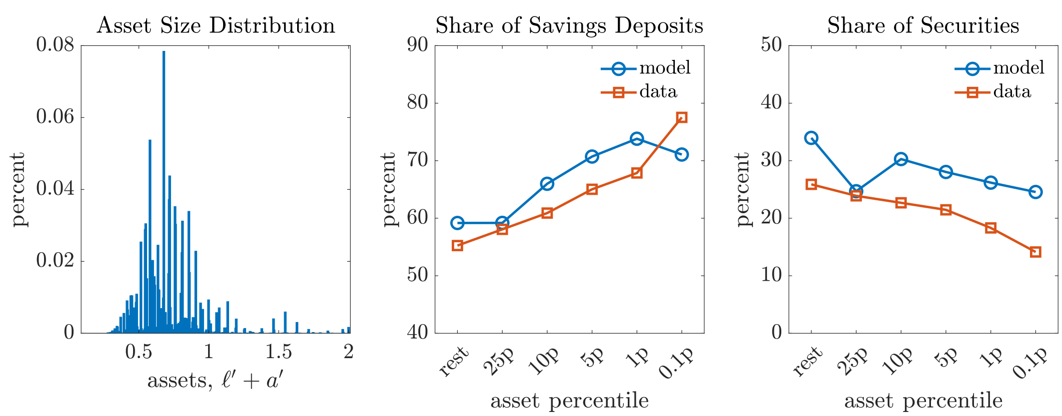
5.4 Solving the model

The banking industry equilibrium is solved by clearing the deposit market. Since the banks have market power over both types of deposits, households observe all deposit prices that banks offer and allocate deposits across bank distribution. This requires mapping the distribution of banks defined over banks' individual states to deposit price dimension. Then, we can retrieve the households' deposit demand functions that the bank faces. Appendix D contains the details of the computational algorithm to solve for the model.

6 Model Validations and Mechanism in Steady State

6.1 Empirical validations of the model

This section provides empirical validations of the model. Although the model is not fully calibrated, our model qualitatively matches the patterns of bank balance sheets across size classes. Figure 7 shows the bank balance sheet composition by the level of total assets, $\ell' + a'$. Based on the left panel of Figure 7, bank size distribution over assets is right-skewed and long right-tail.



Notes: Asset distribution (left), $\tilde{\mu}(\ell' + a')$, is derived by mapping bank states to the choice set of assets, (ℓ', a'). Values for the share of savings deposits and securities in the balance sheet are calculated as a weighted average over asset distribution and expressed on asset percentile as in the data.

Figure 7: Asset distribution and bank balance sheet composition by size

In the middle panel of Figure 7, the model shows that asset-small banks hold a larger share of time deposits in the balance sheet which is consistent with the data. And this share is falling as banks become large. Recall that withdrawal of savings deposit reduces the bank capital next period. And holding securities is required not to sell loans in the middle of the period. Investment in loans that banks can raise their future net worth is feasible with a higher share of time deposits. As banks become large, they substitute time deposits for more savings deposits. Since the interest rate for savings deposits is lower than time deposits, it is desirable. Moreover, the type-specific withdrawal risk contributes to this pattern. For example, medium-sized banks face a lower and less dispersed withdrawal risk for a unit of savings deposit than small banks. This enables medium-sized banks to increase the share of savings deposits to reduce the average funding cost while maintaining or even reducing the securities holding. The share of securities in the asset side of the balance sheet is falling in size on the right panel of Figure 7. Despite a large share of savings deposits, a lower withdrawal risk for a unit of savings deposits in a large bank reduces an incentive to invest in securities. This pattern is also empirically observed, which features the same higher share of securities among small banks and a lower share in large banks.

7 Aggregate Role of Bank Deposit Mix

This section provides an aggregate implication of bank deposit structure. As we see in Figure 7, deposit mix choice affects the asset portfolio at the bank level by considering the relative cost of two types of deposits and withdrawal risk. To further understand the role of deposit structure in loan creation and demand for liquid securities and aggregate outcomes, we consider an experiment of steady state comparison.

7.1 What is the effect of a low risk-free interest rate regime?

Aggregate savings deposit shares in the U.S. deposit market largely increased after the Great Recession (Figure 1) reaching roughly 90 percent of total deposits. One of the explanations for this trend would be a declining interest rate. Based on the portfolio theory of the household problem, a low interest rate reduces the spread between savings and time deposits. The lowered opportunity cost of holding savings deposits allows households to hold more savings deposits. Monetary policy during the Great Recession drove interest rates to levels lower than those before the crisis, and they have not returned to their pre-crisis levels within our sample period.

To emulate the rise in the share of savings deposits with a lowered interest rate, we consider an alternative economy's steady state with an annual risk-free rate at 1% keeping the loan spread the same. This economy is called *Low IR*.

Variable		**Baseline**	**Low IR**
Assets		0.729	0.614
	Loans	0.586	0.413
	Securities	0.143	0.201
	Share of Loans (%)	80.40	67.33
Liabilities		0.675	0.578
	Savings Deposit	0.377	0.485
	Time Deposit	0.298	0.093
	Share of Savings (%)	55.81	83.92
Interest Rates			
	Savings Deposit	0.1%	0.1%
	Time Deposit	1.18%	0.14%
Net Dividend		0.007	0.002
Bank Leverage		0.926	0.941
Liquidity Service		0.882	0.893
Consumption		0.300	0.299
Tax		0.004	0.001

Table 3: Steady State Aggregates: baseline vs. Low IR

Comparing the steady state of baseline with a low risk-free rate economy in Table 3, the spread between savings and time deposit is decreased. The lower opportunity cost of savings deposits induces households to save less with time deposits. Banks can now avoid withdrawal risk by issuing time deposits with lower interest payments. The foregone interest income of holding government bonds to maturity when meeting the withdrawal request in the interim period is smaller than the baseline economy, which allows a bank to hold more savings deposits. In other words, the cost of withdrawal risk in terms of loss in bank capital is reduced in the low interest rate economy. In aggregate, we observe that time deposits fall and savings deposits rise and then the share of savings deposits in the economy rises by 28%p.

In the asset side of the economy, banks can purchase fewer securities because the market value of a unit of securities is higher when it is sold and because some banks hold more time deposits as it becomes cheaper. In aggregate, however, the demand for securities rises as banks have more amount of savings deposits being exposed to withdrawal risk. In response to this, loan supply is decreased. Despite the low risk-free interest rate reducing the cost of withdrawal risk and making time deposits cheaper, a higher share of savings deposits put more deposits in the banking sector into withdrawal risk and banks increase the demand for liquid securities to insure against the sale of loans in response to withdrawal. Hence, the loan supply in the aggregate economy is reduced in the long run by 13%p.

The result of aggregate loan supply in the low risk-free rate economy is consistent with Supera (2021). A larger share of savings deposits puts higher withdrawals and reduces the supply of illiquid loans. Note that the result in Table 3 is the economy in the long run. The usual bank lending channel of monetary policy would have the opposite effect on loan supply and it is a short-run analysis. The model considers heterogeneous banks and the real effect of withdrawal shock on bank capital. Withdrawal risk per unit of savings deposits differs across size classes. Low risk-free rates lessen the liquidity risk for all banks and cheaper time deposits help small banks to avoid withdrawal risk, hence the bank capital in *low IR* can be improved. On the other hand, a decrease in the share of loans in the banking sector implies a lower return on the bank's asset portfolio, which deteriorates the bank's net worth. All in all, a rise in demand for securities crowds out loan supply in the economy because more dollar amounts of savings deposits in the economy are exposed to the withdrawal risk and banks exhibit higher leverage.

7.2 State-dependent withdrawal risk and financial stability

Previously, withdrawal shock is estimated with data and the distribution is constant for each type of bank. One might think that this withdrawal shock distribution can vary over time and it depends on the aggregate state. Since the model has no aggregate uncertainty, we instead compare two steady states with different withdrawal shock distributions with the baseline economy. The distinguishing feature of Figure 6 is that the medium-sized banks face the withdrawal risk as much as small banks although their asset sizes, on average, are closer to the large bank group. Therefore, medium-sized banks are particularly important in explaining the liquidity risk in the banking sector and its aggregate outcome. To study the effect of state-dependent withdrawal risk, we consider an alternative economy where medium-sized banks face the same withdrawal shock distribution as large banks. This economy is called SD (state-dependent) withdrawal.

Table 4 shows the aggregate variables of steady state of two economies. A reduced withdrawal risk for medium-sized banks allows a larger inflows of savings deposits in the economy. The rise in share of savings deposits by 5%p does not lead to a decrease in loan supply as in the previous experiment with low risk-free interest rate. Medium-sized banks now demand less securities to insure against withdrawal risk and the share of loans in the economy increases by 2%p. In terms of dollar amounts, aggregate bank's balance sheet is expanded. A higher share of loans in the balance sheet enables medium-sized banks to accumulate bank capital more easily as loan returns higher than securities.

Conversely, as the medium-sized banks are exposed to a higher liquidity risk than large banks in the baseline economy, it raises the equilibrium interest rate for time deposits which deteriorates profitability of banks. A larger liquidity risk compared with *SD withdrawal* requires medium-sized banks to hold more liquid assets and the loan supply is decreased. This result has an implication for the financial stability. As the medium-sized banks that are relevant for aggregate outcome are exposed to relatively large withdrawal risk, there is a decrease in loan supply in the aggregate economy.

The effect is not ended to medium-sized banks. Equilibrium effect on deposit market lowers the interest rates for time deposits by 8 bp. Although the effect on the aggregate economy is not large, small banks which are mostly depending on time deposits would face a lower average funding cost.

Variable		**Baseline**	**SD Withdrawal**
Assets		0.729	0.752
	Loans	0.586	0.618
	Securities	0.143	0.134
	Share of Loans (%)	80.40	82.23
Liabilities		0.675	0.695
	Savings Deposit	0.377	0.419
	Time Deposit	0.298	0.276
	Share of Savings (%)	55.81	60.27
Interest Rates			
	Savings Deposit	0.1%	0.1%
	Time Deposit	1.18%	1.10%
Net Dividend		0.007	0.008
Bank Leverage		0.926	0.924
Liquidity Service		0.882	0.885
Consumption		0.300	0.301
Tax		0.004	0.004

Table 4: Steady State Aggregates: baseline vs. SD withdrawal

Next, we examine the short-term impact of withdrawal risk and varying liquidity risk in the banking sector. The experiment involves an initial 10 percent loss in bank net worth at the impact date. By period 5, the net worth shock returns to zero, with the shock size decreasing by 2.5 percentage points each period after the initial impact. An alternative economy with a higher withdrawal risk is considered for comparison, called *High Risk (HR)*. This economy features 10 percent higher withdrawal shocks for all banks.

Figure 8 shows the impulse response to an adverse aggregate shock to bank net worth. As banks in the economy have market power over their deposits, interest rates for deposits fall. This helps to achieve a higher net interest margin and recover bank capital over time. In response to loss in bank capital, total assets and deposits are reduced relative to the level of steady states. Comparing the response from the baseline economy with *high risk* economy, we see that a higher withdrawal risk and then an increased liquidity risk in the banking sector amplify the adverse net worth shock in the banking sector. Banks in a higher withdrawal risk economy set a lower interest rate for deposits to maintain high average net interest margins but the profitability overall gets worse which is reflected in the net worth response.

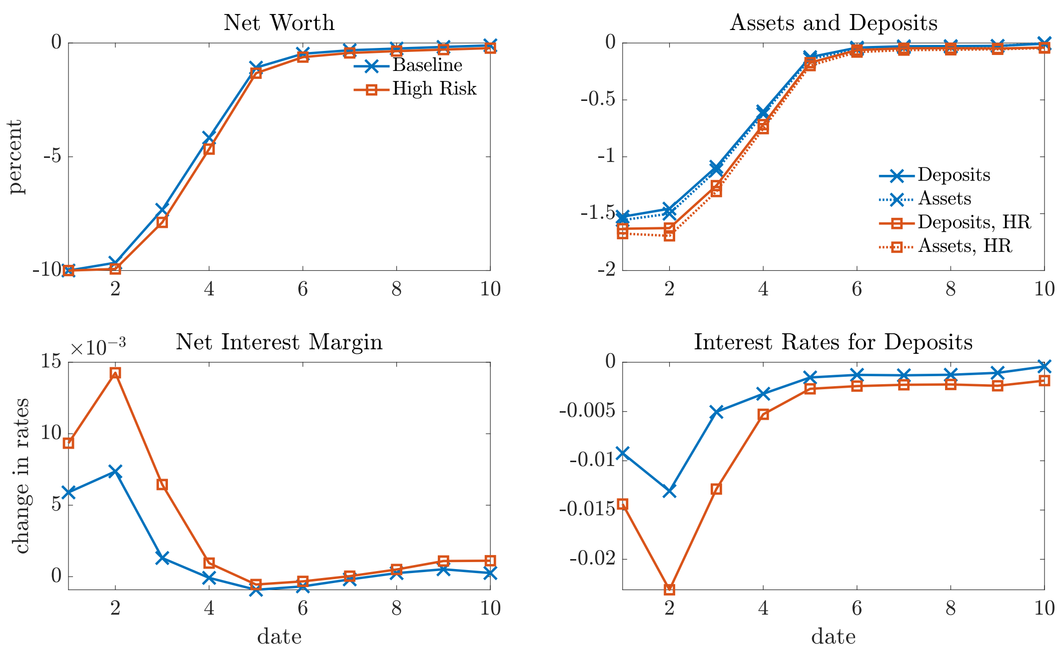


Figure 8: Impulse Response to Aggregate Shock to Bank Net Worth

In Figure 9, I see the impulse response to the same net worth shock but for each asset and deposit component. Looking at the first column, loans are more responsive to the negative net worth shock in both economies which occurs because of the capital requirement constraint. Banks now face a lower net worth and are not reducing securities much so as not to violate the capital requirement. The fall in loan supply is amplified by a higher risk of withdrawal. This is the case because a higher withdrawal risk adds a stronger motive to hold liquid assets when the bank's capital becomes small. So the banking sector features "flight-to-safety" or "flight-to-liquidity" replacing securities as the asset in the balance sheet with illiquid loans.

A more responsive loan supply implies a rise in the share of securities in the balance sheet. This lets banks lower the issuance of time deposits more in response to shock. As in the second column of the figure, time deposits are reduced more than savings deposits, which raises the share of savings deposits in the balance sheet. This is helpful for banks to recover their profitability provided that savings deposits pay lower interest rates. Banks in a higher withdrawal risk economy reduce the issuance of savings deposits less and show a higher share of savings deposits over time than banks in the baseline model. Although per unit withdrawal risk is higher, securities still return higher than deposits, and lowering the average funding cost is necessary to recover bank capital.

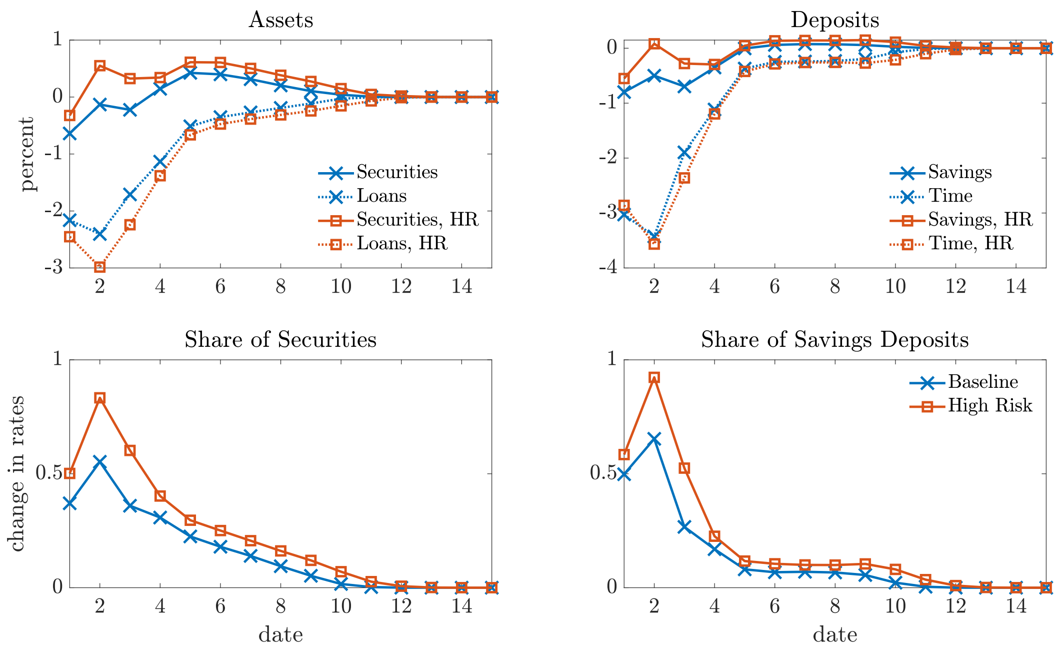


Figure 9: Impulse Response to Aggregate Shock to Bank Net Worth, Balance Sheet Composition

7.3 Policy implications: external liquidity provision

In this subsection, I discuss the policy implication of heterogeneous bank liquidity risk which stems from different withdrawal risks across different sizes of banks. In order to preserve financial stability, the Federal Reserve (Fed) acts as a liquidity provider for the banking industry as a form of discount window on top of various banking regulations. As the central bank provides liquidity to banks that face an unexpected or a large outflow of short-term financial obligations, bank assets do not need to be sold before maturity with a significant discount. In this way, banks can persistently originate loans in the economy. To quantitatively assess the effect of having a central bank as a liquidity provider for the banking industry, I consider a policy rule for liquidity provision in the economy and compare the short-run dynamics with the baseline economy where there is no central bank.

Assume that the central bank is integrated with the government. When the shock hits the economy, the central bank starts to provide liquidity in the banking sector so that 10 percent of actual withdrawals in each bank are covered by borrowings from the central bank during the presence of an adverse shock. This borrowing must be repaid at the end of the period when the asset returns are realized. Denote the central bank's spending for a bank as $\widehat{a}' = 0.1 \times \theta \tilde{d}'_S / q_a$. The government budget constraint changes to

$$\int \tilde{a}' \mathrm{d}\mu + \int q_a \widehat{a}' \mathrm{d}\mu = \tau' + \int q'_a \tilde{a}'' \mathrm{d}\mu + \int \widehat{a}' \mathrm{d}\mu$$

In a steady state, the government's budget balance implies

$$\tau = \int (1 - q_a) \{\tilde{a}' - \widehat{a}'\} \mathrm{d}\mu$$

On the bank side, the effective withdrawal request is $0.9 \times \theta \tilde{d}'_S$ and net worth for the next period includes the repayment of borrowing from the central bank, so it becomes

$$n' = \ell' + a' - d'_S - d'_T - \widehat{a}'$$

Again, although the repayment of securities borrowed from the central bank can reduce the bank capital in the next period, liquidity provision helps to lower the effective withdrawal of savings deposits and so the bank's demand for liquid assets at the beginning of the period can be lowered. Fixing the amount of deposits issued, a lowered incentive to purchase liquid assets to insure against withdrawal risk frees room for loan supply.

Figure 10 shows the impulse response to adverse net worth shock. An alternative economy with a central bank as a liquidity provider is denoted as *LP*. Comparing these two economies, bank capital is more rapidly recovered as banks do need to decrease much of the loan supply. Time deposit issuance is more decreased than the baseline economy as a fraction of withdrawal is serviced by borrowing from the central banks. So the bank would set even lower interest rates for deposits to lower the funding cost which additionally helps to recover the net worth.

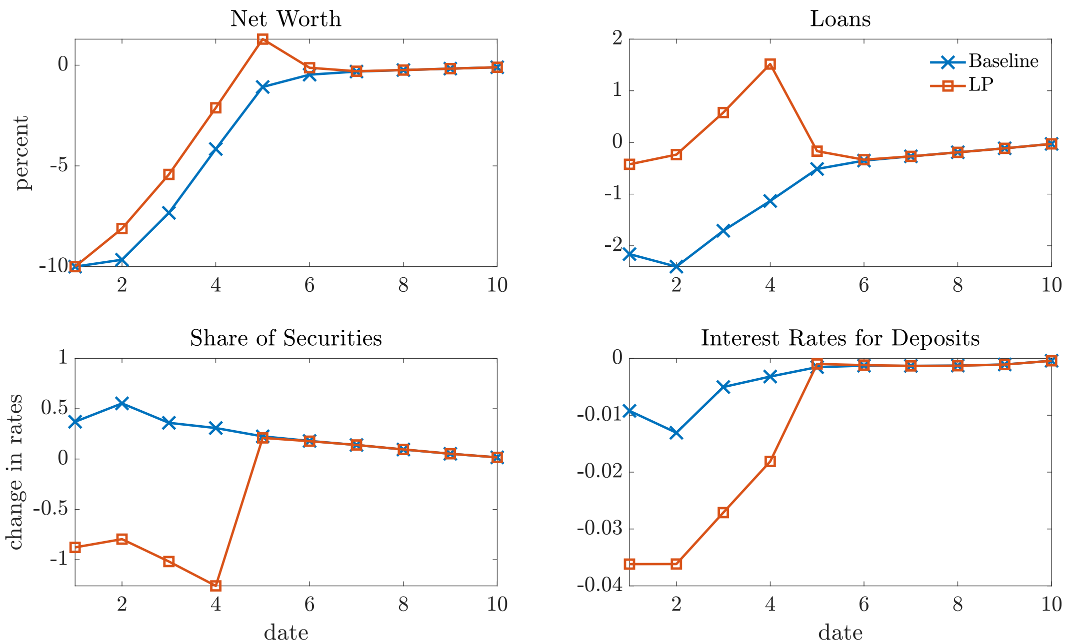


Figure 10: Impulse Response to Aggregate Shock to Bank Net Worth, Liquidity Provision

I further inspect the propagation mechanism by looking at the responses across different size groups. The first row of Figure 11 shows the response of securities and loans in two economies across different size groups. In the second row, I compute the change in response between the baseline and *LP*. The central bank's liquidity provision has a distributional effect. Large and medium-sized banks are more responsive and can reduce the demand for securities more than small banks. And banks can reduce the loan supply less with liquidity provision by the central bank. Among size groups, medium-sized banks show the strongest response in loan supply. They actually increase the loan supply while for large banks the effect on the increase in loans is somewhat limited although their response in decreasing in securities demand is the largest. The assumed policy rule of the discount window is particularly effective for medium-sized banks that are as large as large banks but as risky as small banks.

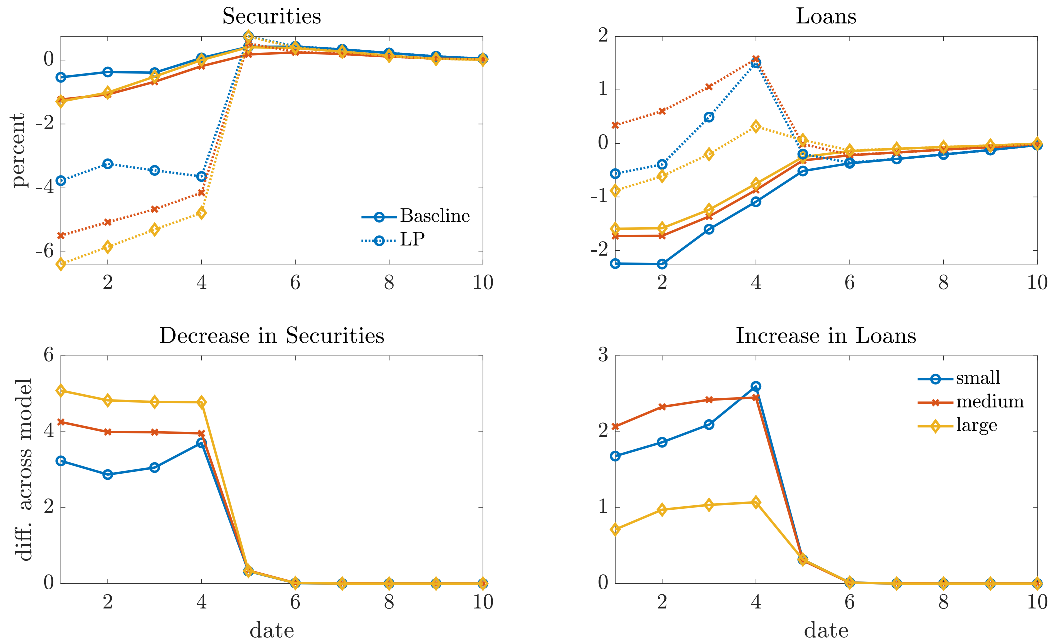


Figure 11: Responses in Assets Across Different Size Groups

Based on the findings in this subsection, I plan to discuss further the optimal level of discount window or size-dependent policy rules (bilateral contract between a bank and the central bank considering the bank size). On top of that, the model can be extended to include the liquidity requirement proposed by Basel III in the bank problem and see how this particular regulation changes the bank's incentive in holding liquid assets, deposit mix, and liquidity risk of the banking sector. The same discount window by central bank in the presence of such regulation is also an interesting policy evaluation using the model.

8 Concluding Remarks

This paper studies the effect of heterogeneous bank deposit mix and liquidity risk on financial stability in the aggregate economy. Bank deposits are the main source of funds for banks to operate liquidity transformation. Most banks mix different types of deposit products rather than issuing a single type. Savings deposits are relatively volatile but pay lower interest rates than time deposits. Large banks issue savings deposits which take up more than 80% of total deposits in the balance sheet, whereas banks' dependence on time deposits is increasing as banks become smaller. The bottom 90% holds 40% of the total deposits in time deposits on the balance sheet. Looking at the outflows of savings deposits, large banks face smaller outflows on average and less frequent large outflows than small banks. With the free withdrawal of savings deposits, banks optimally choose a deposit mix considering the tradeoff between the interest cost of deposits and the degree of stability in funding flows.

We build an industry equilibrium model of heterogeneous banks with liquidity risk. Savings deposits are a cheaper source of external finance for banks at the expense of withdrawal risk. Withdrawal of savings deposits incurs asset re-balancing and potentially reduces the bank capital and this provides an incentive for a bank to demand liquid assets. Banks can avoid it by issuing costly time deposits. Because the banking industry is highly concentrated and large banks have the advantage of diversifying deposit sources, withdrawal risk is assumed to be size-dependent. Loans are illiquid in the sense that they accompany large discounts when sold before maturity. Therefore, banks match liquidity across assets and liabilities.

The model successfully generates qualitative features of bank balance sheet patterns for deposit composition and asset composition across different sizes. Withdrawal risk depending on the type of bank is important to produce a declining share of securities and a rising share of savings deposits. A steady-state comparison with a low risk-free rate economy is used to check the model mechanism. A low risk-free rate reduces the average funding cost of banks and the loss of bank capital associated with withdrawal is mitigated. However, a decline in the opportunity cost of holding savings deposits for households reduces the equilibrium share of time deposits. Then, the rise of demand for liquid assets crowds out the supply of illiquid loans. In another experiment with state-dependent withdrawal risk, we show the medium-sized banks and how much the change in their withdrawal shock distribution affects the aggregate economy. A higher liquidity risk for medium-sized banks reduces a bank's balance sheet. It reduces the share of savings deposits and loans.

To further study the role of deposit mix and heterogeneous bank liquidity in short-run dynamics, I consider the perfect foresight with an adverse aggregate shock to bank net worth. An increase in bank withdrawal risk amplifies the response of the banking sector so it further reduces the loan supply. The presence of withdrawal shock and capital requirement constraint incentivizes banks to reduce liquid assets less in response to the shock and the loan supply falls.

Lastly, I provide a policy implication of the deposit mix and heterogeneous bank liquidity risk. The central bank provides liquidity when the bank faces large and/or unexpected outflows in liabilities through a discount window. Given the adverse net worth shock, liquidity provision by the central bank accelerates the recovery of the bank's net worth. As the effective withdrawal is reduced, the bank demands fewer securities, and the loan supply does not need to fall as much as the baseline economy. Lowered effective withdrawal risk induces banks to issue more savings deposits and lower the average funding cost. Looking at the response across different size groups, there is a distributional effect of the discount window. Large and medium-sized banks mostly benefited in terms of lowering demand for securities. However, large banks are not very responsive in increasing loan supply from the liquidity provision. Medium-sized banks that are as large as large banks and as risky as small banks are the most responsive and increase the loan supply the most. The model can be used to study banking regulation, especially for liquidity requirements proposed in Basel III. While studying the effect of the regulation on the banking industry and aggregate consequences, size-dependent liquidity risk can extend the understanding of how differently such a policy can affect banks and form aggregate outcomes.

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A Proofs

A.1 Proof of Proposition 1

Proof. Consider first the case that only government bonds are enough to serve the withdrawal request.

$$\begin{aligned}
n_0' &= \tilde{\ell}' + \tilde{a}' - \frac{\theta \tilde{d}_S'}{q_a} - (1-\theta)\tilde{d}_S' - \tilde{d}_T' \\
&= \tilde{\ell}' + \tilde{a}' - \tilde{d}_S' - \tilde{d}_T' - \left(\frac{1}{q_a} - 1\right)\theta \tilde{d}_S' \\
&= \overline{n}' - \left(\frac{1}{q_a} - 1\right)\theta \tilde{d}_S' < \overline{n}' \quad \text{if} \quad q_a < 1
\end{aligned}$$

where $\overline{n}' \equiv \tilde{\ell}' + \tilde{a}' - \tilde{d}_S' - \tilde{d}_T'$ defines the maximal net worth in the next period that a bank can achieve with a balance sheet choice, $\{\tilde{\ell}', \tilde{a}', \tilde{d}_S', \tilde{d}_T'\}$.

The difference in net worth comes from the foregone interest income associated with the government bonds that are used for serving the withdrawal before its maturity. The loss is positive when the net risk-free interest rate is greater than zero.

Second, when loans are additionally needed to be sold with a discount

$$\begin{aligned}
n_1' &= \tilde{\ell}' - \frac{\theta \tilde{d}_S' - q_a \tilde{a}'}{\omega q_\ell} - (1-\theta)\tilde{d}_S' - \tilde{d}_T' \\
&= \tilde{\ell}' + \tilde{a}' - \tilde{d}_S' - \tilde{d}_T' - \left(\frac{1}{\omega q_\ell} - 1\right)\theta \tilde{d}_S' + \left(\frac{q_a}{\omega q_\ell} - 1\right)\tilde{a}' \\
&< \overline{n}' - \left(\frac{1}{\omega q_\ell} - 1\right) q_a \tilde{a}' + \left(\frac{q_a}{\omega q_\ell} - 1\right)\tilde{a}' \quad (\because \quad \theta d_S' > q_a a') \ \& \ \text{if} \ \ \omega q_\ell < 1 \\
&= \overline{n}' + q_a \tilde{a}' - \tilde{a}' = \overline{n}' - (1-q_a)\tilde{a}' < \overline{n}' \quad \text{if} \quad q_a < 1
\end{aligned}$$

$\square$

A.2 Proof of Corollary 1

Proof. Recall the net worth in the next period, n_0' and n_1' are from the Proof of Proposition 1.

$$n_0' = \overline{n}' - \left(\frac{1}{q_a} - 1\right)\theta \tilde{d}_S'$$
$$n_1' = \tilde{\ell}' + \tilde{a}' - \tilde{d}_S' - \tilde{d}_T' - \left(\frac{1}{\omega q_\ell} - 1\right)\theta \tilde{d}_S' + \left(\frac{q_a}{\omega q_\ell} - 1\right)\tilde{a}'$$

An extra unit of withdrawal has an adverse effect on the future net worth by $\frac{\partial n_0'}{\partial(\theta \tilde{d}_S')}$ and $\frac{\partial n_1'}{\partial \theta \tilde{d}_S'}$.

$$\frac{\partial n_0'}{\partial(\theta \tilde{d}_S')} = -\frac{1}{q_a}$$
$$\frac{\partial n_1'}{\partial(\theta \tilde{d}_S')} = -\frac{1}{\omega q_\ell}$$

Thus, $\left|\frac{\partial n_0'}{\partial(\theta \tilde{d}_S')}\right| < \left|\frac{\partial n_1'}{\partial(\theta \tilde{d}_S')}\right|$ if and only if $\omega q_\ell < q_a < 1$. □

B Derivations

B.1 Household's optimal policies

Suppose μ is the Lagrange multiplier for 2. Consider the optimality conditions for the inner-most CES composite first (i.e., the allocation of type of deposits in bank j). First-order conditions are

$$
\begin{aligned}
[C] &: Z^{-\gamma} Z^{\frac{1}{\eta}} \lambda_C^{\frac{1}{\eta}} C^{-\frac{1}{\eta}} = \mu \\
[d'_{S,j}] &: Z^{-\gamma} Z^{\frac{1}{\eta}} (1-\lambda_C)^{\frac{1}{\eta}} L^{-\frac{1}{\eta}} L^{\frac{1}{\nu}} \delta_j^{\frac{1}{\nu}} L_j^{-\frac{1}{\nu}} L_j^{\frac{1}{\epsilon}} \lambda_S^{\frac{1}{\epsilon}} (d'_{S,j})^{-\frac{1}{\epsilon}} + \beta \mathbb{E}\left[\frac{\partial V^H(D'_S, D'_T; \mu'_B)}{\partial d'_{S,j}}\right] = q_{S,j}\mu \\
[d'_{T,j}] &: Z^{-\gamma} Z^{\frac{1}{\eta}} (1-\lambda_C)^{\frac{1}{\eta}} L^{-\frac{1}{\eta}} L^{\frac{1}{\nu}} \delta_j^{\frac{1}{\nu}} L_j^{-\frac{1}{\nu}} L_j^{\frac{1}{\epsilon}} (1-\lambda_S)^{\frac{1}{\epsilon}} (d'_{T,j})^{-\frac{1}{\epsilon}} + \beta \mathbb{E}\left[\frac{\partial V^H(D'_S, D'_T; \mu'_B)}{\partial d'_{T,j}}\right] = q_{T,j}\mu
\end{aligned}
$$

Envelope condition is $\frac{\partial V^H(D_S, D_T; \mu'_B)}{\partial d_{S,j}} = \frac{\partial V^H(D_S, D_T; \mu'_B)}{\partial d_{T,j}} = \mu$. In the stationary economy, $\mu' = \mu$ holds and combining the first-order conditions for deposits lead to

$$
\begin{aligned}
\left(\frac{\lambda_S}{1-\lambda_S}\right)^{\frac{1}{\epsilon}} (d'_{S,j})^{-\frac{1}{\epsilon}} \left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right) &= (d'_{T,j})^{-\frac{1}{\epsilon}} \\
\Rightarrow d'_{T,j} &= \frac{1-\lambda_S}{\lambda_S} \left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon} d'_{S,j}
\end{aligned} \tag{20}
$$

Consider the total expenditure for $d'_{S,j}$ and $d'_{T,j}$ for bank j and denote it as $E_j \equiv q_{S,j} d'_{S,j} + q_{T,j} d'_{T,j}$. Using the equation 20,

$$
\left(q_{S,j} + \frac{1-\lambda_S}{\lambda_S} \left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon} q_{T,j}\right) d'_{S,j} = E_j \tag{21}
$$

Then, we can solve for $d'_{S,j}$ and $d'_{T,j}$ as function of $\{q_{S,j}, q_{T,j}, E_j\}$ and model parameters

$$\begin{aligned}
d'^{*}_{S,j}(q_{S,j}, q_{T,j}, E_j) &= \frac{\lambda_S}{\lambda_S q_{S,j} + (1-\lambda_S)\left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon} q_{T,j}} E_j \\
&= \frac{\lambda_S \left(q_{S,j}-\beta\right)^{-\epsilon}}{\lambda_S \left(q_{S,j}-\beta\right)^{-\epsilon} q_{S,j} + (1-\lambda_S)\left(q_{T,j}-\beta\right)^{-\epsilon} q_{T,j}} E_j \qquad (22) \\
d'^{*}_{T,j}(q_{S,j}, q_{T,j}, E_j) &= \frac{(1-\lambda_S)\left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon}}{\lambda_S q_{S,j} + (1-\lambda_S)\left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon} q_{T,j}} E_j \\
&= \frac{(1-\lambda_S)\left(q_{T,j}-\beta\right)^{-\epsilon}}{\lambda_S \left(q_{S,j}-\beta\right)^{-\epsilon} q_{S,j} + (1-\lambda_S)\left(q_{T,j}-\beta\right)^{-\epsilon} q_{T,j}} E_j \qquad (23)
\end{aligned}$$

To achieve the price index of liquidity service from the bank j, substitute the equations 22 and 23 into 5.

$$L_j = \frac{(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon})^{\frac{\epsilon}{\epsilon-1}}}{\lambda_S(q_{S,j}-\beta)^{-\epsilon} q_{S,j} + (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon} q_{T,j}} E_j$$

Define the price index as $P_{L,j} \equiv E_j|_{L_j=1}$. Then,

$$P_{L,j} = \frac{\lambda_S(q_{S,j}-\beta)^{-\epsilon} q_{S,j} + (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon} q_{T,j}}{(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon})^{\frac{\epsilon}{\epsilon-1}}}$$

Using the price index, we can re-express the optimal deposit demand functions as

$$\begin{aligned}
d'^{*}_{S,j}(q_{S,j}, q_{T,j}, E_j) &= \frac{1}{P_{L,j}} \left(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} \lambda_S(q_{S,j}-\beta)^{-\epsilon} E_j \\
d'^{*}_{T,j}(q_{S,j}, q_{T,j}, E_j) &= \frac{1}{P_{L,j}} \left(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon} E_j
\end{aligned}$$

Substituting these back to 5 confirms that $P_{L,j}$ is indeed the price index for the aggregate liquidity service for bank j, $P_{L,j}L_j = E_j = q_{S,j}d'_{S,j} + q_{T,j}d'_{T,j}$. And the deposit demand function is expressed with bank-level liquidity service.

$$d'^{*}_{S,j}(q_{S,j}, q_{T,j}, L_j) = \lambda_S(q_{S,j}-\beta)^{-\epsilon} \left(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} L_j \qquad (24)$$

$$d'^{*}_{T,j}(q_{S,j}, q_{T,j}, L_j) = (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon} \left(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} L_j \qquad (25)$$

Now, let's consider the CES composite in the middle (i.e., allocation of liquidity services across banks). Using the price index for liquidity service for bank j, we can re-write the household's budget constraint as

$$C + \int \{P_{L,j} L_j\} \, \mathrm{d}j \leq w + D_S + D_T + \Pi - \tau$$

Let's consider the first-order condition for L_j.

$$Z^{-\gamma} Z^{\frac{1}{\eta}} (1 - \lambda_C)^{\frac{1}{\eta}} L^{-\frac{1}{\eta}} L^{\frac{1}{\nu}} \delta_j^{\frac{1}{\nu}} L_j^{-\frac{1}{\nu}} = P_{L,j} \mu$$

Note that allocating liquidity service over the distribution of banks is a static choice, so the optimality condition does not involve continuation value. Considering the same first-order condition for bank $i \neq j$, we have

$$\begin{aligned} \left(\frac{\delta_j}{\delta_i}\right)^{\frac{1}{\nu}} L_j^{-\frac{1}{\nu}} &= \left(\frac{P_{L,j}}{P_{L,i}}\right) L_i^{-\frac{1}{\nu}} \\ \Longrightarrow L_i &= \left(\frac{\delta_i}{\delta_j}\right) \left(\frac{P_{L,i}}{P_{L,j}}\right)^{-\nu} L_j \end{aligned} \tag{26}$$

Define $E_L \equiv \int P_{L,i} L_i \mathrm{d}i = \int E_i \mathrm{d}i$ to be the total expenditure for deposits across all banks. Substituting 26 into the definition of E_L yields

$$\begin{aligned} E_L = \int P_{L,i} \left(\frac{\delta_i}{\delta_j}\right) \left(\frac{P_{L,i}}{P_{L,j}}\right)^{-\nu} L_j \mathrm{d}i &= \left(\int \delta_i P_{L,i}^{1-\nu} \mathrm{d}i\right) \frac{P_{L,j}^{\nu} L_j}{\delta_j} \\ \Longrightarrow L_j = \frac{\delta_j}{P_{L,j}^{\nu}} \cdot \frac{E_L}{\int \delta_i P_{L,i}^{1-\nu} \mathrm{d}i} \end{aligned} \tag{27}$$

use 27 and back into 4:

$$L = \left(\int \delta_j^{\frac{1}{\nu}} L_j^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} = \left[\int \delta_j^{\frac{1}{\nu}} \delta_j^{\frac{\nu-1}{\nu}} P_{L,j}^{1-\nu} \mathrm{d}j\right]^{\frac{\nu}{\nu-1}} \frac{E_L}{\int \delta_i P_{L,i}^{1-\nu} \mathrm{d}i}$$

The price index can be defined as $P_L \equiv E_L | L = 1$. Then,

$$P_L = \left[\int \delta_j P_{L,j}^{1-\nu} \mathrm{d}j \right]^{\frac{1}{1-\nu}}$$

so, $P_L L = E_L = \int P_{L,i} L_i \mathrm{d}i = \int E_i \mathrm{d}i = \int \left\{ q_{S,i} d'_{S,i} + q_{T,i} d'_{T,i} \right\} \mathrm{d}i$. Using this,

$$L_j^*(P_{L,j}, P_L, L) = \delta_j P_{L,j}^{-\nu} \frac{E_L}{P_L^{1-\nu}} = \delta_j \left(\frac{P_{L,j}}{P_L} \right)^{-\nu} L \tag{28}$$

Now the budget constraint can be re-expressed as

$$C + P_L L \leq w + D_S + D_T + \Pi - \tau$$

Note that the choice of liquidity service is a static decision as if L is another type of consumption goods. Household's intertemporal choice is already considered in the inner CES problem and the outer problem cares only about allocating the available resources to consumption and aggregate liquidity service.

The first-order conditions are

$$\begin{aligned} [C]: &\ \ Z^{-\gamma} Z^{\frac{1}{\eta}} \lambda_C^{\frac{1}{\eta}} C^{-\frac{1}{\eta}} = \mu \\ [L]: &\ \ Z^{-\gamma} Z^{\frac{1}{\eta}} (1 - \lambda_C)^{\frac{1}{\eta}} L^{-\frac{1}{\eta}} = P_L \mu \end{aligned}$$

Combining them together yields

$$\begin{aligned} P_L \left(\frac{\lambda_C}{1 - \lambda_C} \right)^{\frac{1}{\eta}} C^{-\frac{1}{\eta}} &= L^{-\frac{1}{\eta}} \\ \Rightarrow L &= \left(\frac{1 - \lambda_C}{\lambda_C} \right) P_L^{-\eta} C \end{aligned}$$

As the utility function is monotonically increasing in both consumption and total liquidity, budget constraint binds at the optimum. Substituting back to the budget constraint leads to

$$\left(1+\left(\frac{1-\lambda_C}{\lambda_C}\right)P_L^{1-\eta}\right)C = w + D_S + D_T + \Pi - \tau$$

To simplify the notation, let $\overline{w} \equiv w + \Pi - \tau$. We can solve for the optimal consumption and liquidity service as functions of prices and $(\overline{w}, D_S, D_T)$.

$$C = \frac{\lambda_C}{\lambda_C + (1-\lambda_C)P_L^{1-\eta}}(\overline{w} + D_S + D_T)$$
$$L = \frac{(1-\lambda_C)P_L^{-\eta}}{\lambda_C + (1-\lambda_C)P_L^{1-\eta}}(\overline{w} + D_S + D_T)$$

By substituting to the CES aggregator, Z, the price index, $P_Z \equiv (\overline{w} + D_S + D_T)|_{Z=1}$, is derived.

$$P_Z = (\lambda_C + (1-\lambda_C)P_L^{1-\eta})^{\frac{1}{1-\eta}}$$

Using this price index, demand functions for consumption and liquidity can be re-expressed as

$$C = \frac{1}{P_Z}\lambda_C\left(\frac{1}{P_Z}\right)^{-\eta}(\overline{w} + D_S + D_T)$$
$$L = \frac{1}{P_Z}(1-\lambda_C)\left(\frac{P_L}{P_Z}\right)^{-\eta}(\overline{w} + D_S + D_T)$$

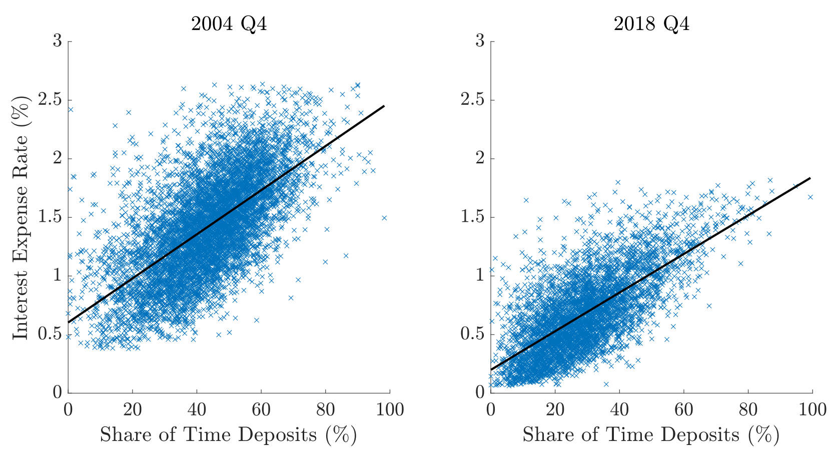
Similar to the inner CES problem, substituting these demand functions into 3 again confirms $P_Z Z = \overline{w} + D_S + D_T$. Then, optimal consumption and liquidity demand functions are

$$C(P_Z, Z) = \lambda_C\left(\frac{1}{P_Z}\right)^{-\eta}Z$$
$$L(P_L, P_Z, Z) = (1-\lambda_C)\left(\frac{P_L}{P_Z}\right)^{-\eta}Z$$

C Regression analysis: share of time deposits and profitability

C.1 Deposit share and bank profitability

A higher dependence on time deposits among small-sized banks leaves a question about the adverse effect on their profitability given that sourcing funds using time deposits is more expensive as documented earlier. Although deposits are the primary source of funding for the banking business, banks can borrow in other forms of liabilities or accumulate capital to compensate for such higher costs associated with time deposits. Scatter plots in Figure 12 show the relationship between the share of time deposits in the balance sheet and the interest expense rates for two specific sample periods. They show a clear positive correlation implying that banks with a higher share of time deposits also face a higher interest expense rate. This correlation is confirmed again with the regression analysis with a full sample range



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1987 to 2020. Every aggregation is asset-weighted. Scatter plot is provided for two periods (2004 Q4 and 2018 Q4), and this pattern holds for other periods).

Figure 12: The share of time deposits and the interest expense rate

controlling bank size, other liabilities, and equity ratio.

The relationship between the share of time deposits in the individual bank's balance sheet and its profitability is studied with a regression analysis. Consider the following specification,

$$Y_{b,t} = \alpha_0 + \beta X_{b,t} + \gamma \mathbf{Z}_{b,t} + \alpha_b + \alpha_t + \epsilon_{b,t} \tag{29}$$

where the subscript b is the name of the bank. For the profitability, we use bank interest expense rate ($= \frac{\text{interest expense}}{\text{asset}}$) which is denoted as $Y_{b,t}$ in the regression equation. $X_{b,t}$ is the share of time deposits. $\mathbf{Z}_{b,t}$ is the vector of control variables. For the controls, we use bank equity ratio, non-deposit liability ratio, total assets, and lagged variable of X and $\mathbf{Z}$. Regression also includes bank and time-fixed effects. Clustered standard errors are computed in the following Table 5. Each column of the table differs by the control variables. We consider the second column without lagged control variables as the baseline result. $\beta = 0.0229$ implies that a 1%p rise in the share of time deposits raises the interest expense rate by 2.3bps. This number is comparable with the coefficient for the equity ratio. By this regression result, we can conclude that a 1%p drop in equity ratio can be equivalently compensated by lowering the time deposit share by 1.55%p. Positive and statistically significant $\hat{\beta}$ shows that after controlling the size of the banks and their alternative funding sources, banks with a higher share of time deposits on average face a larger interest expense rate. We interpret this as an unavoidable cost of time deposits even with alternative options of funding, which leads us to consider the potential advantage of holding time deposits regarding managing the flow of liabilities and/or credit supply.

	(1) Model 1	(2) Model 2	(3) Model 3
Time deposit share	0.0251***	0.0229***	0.0147***
	(0.0007)	(0.0006)	(0.0006)
Asset	-0.0004	-0.0002	0.0061*
	(0.0003)	(0.0003)	(0.0025)
Equity ratio		-0.0356***	-0.0325***
		(0.0014)	(0.0028)
Other liabilities		0.0088***	0.0027***
		(0.0005)	(0.0005)
Lag_Time deposit share			0.0088***
			(0.0005)
Lag_asset			-0.0064*
			(0.0026)
Lag_equity ratio			-0.0027
			(0.0023)
Lag_other liabilities			0.0074***
			(0.0006)
Constant	2.0634***	2.4853***	2.4085***
	(0.0302)	(0.0236)	(0.0253)
Bank FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes
N	1239626	1239626	1221149

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 5: Regression analysis

D Numerical algorithm

D.1 Steady state

Let's denote j to summarize state variables for a bank. To solve for the industry equilibrium with deposit market equilibrium,

1. Assume $g^0_{q_S}(j),\ g^0_{q_S}(j),\ \mu^0(j),\ L^0$
   - start with guessed $P^0_{L,j},\ P^0_L,\ L^0_j$ from 14, 15, and 13
2. Solve for the bank's problem to get $g^1_\ell(j),\ g^1_a(j),\ g^1_{q_S}(j),\ g^1_{q_T}(j)$ facing 11 and 12
3. Compute the stationary distribution, $\mu^1(j)$ by 9
4. Calculate
   - implied $P^1_{L,j}, P^1_L, L^1_j$ using $g^1_{q_S}(j),\ g^1_{q_T}(j),$ and, $\mu^1(j)$
   - D_S, D_T, and $\overline{w}$ based on the bank's choices and distribution
   - L^1 from household's problem
5. Iterate until we get $||L^0 - L^1||$ and $||P^0_L - P^1_L||$ to be small enough.
   - iteration with updates on $g^0_{q_S}(j),\ g^0_{q_T}(j),\ \mu^0(j),\ L^0$

D.2 Transitional dynamics

Suppose the total length of transition period is N_T and t denotes the date for each period.

1. Guess the initial set of L^0_t and $P^0_{L,t}$ containing N_T number of L and P_L for each t
2. start with value function at the terminal period, $v_{N_T} = v^0_{N_T}$
3. back solve for bank's optimal policies from $t = N_T$ to $t = 1$
   - produces $\{g_{\ell,t},\ g_{a,t},\ g_{S,t},\ g_{T,t}\}$

- value function, v_t^1

4. forward solve to update the distribution from $t = 1$ to $t = N_T$

   - start with guessed $\mu_1 = \mu_1^0$
   - produces $\mu_t(n, \delta)$
   - implied prices and bank-level liquidity are computed, $P_{L,t}^1$ and $L_{j,t}^1$
   - $D_{S,t}$, $D_{T,t}$, and $\overline{w}_t$ are calculated
   - L_t^1 from the household's problem

5. check the convergence of $||L_t^0 - L_t^1||$ and $||P_{L,t}^0 - P_{L,t}^1||$ and iterate with updated L_t^0, $P_{L,t}^0$, v_t^0, and μ_t^0

E Data description

E.1 Call Reports

Bank-level data used in the paper is from Consolidated Reports of Condition and Income (so-called Call Reports). The sample is restricted to include U.S. commercial banks (domestic only) located within the 50 states and the District of Columbia. The sample period covers from 1984 Q1 to 2021 Q2.

All variables are deflated with the Consumer Price Index (CPI) to exclude the effect of trends in the economy's price level. To rule out unusual variations in the bank's balance sheet arising from bank entry and exit, I exclude the first and the last observations of each bank. The following table shows the definition of the variables used in the paper.

Name	Variable	Coverage
Assets	RCFD3368	1978/12-
Securities	RCFD0390	1984/03-1993/12
	RCFD1754 + RCFD1773	1994/03-
Liabilities	RCFD2948	1976/03-
Demand Deposits	RCON2210	1976/03-
Time Deposits	RCON2604 + RCON6648	1984/03-2009/12
	RCONJ474 + RCONJ473 + RCON6648	2010/03-
Savings Deposits	RCON2215 + RCON2389	1984/03-
	RCON2215 + RCON0352 + RCON6810	1987/03-
Deposits	Savings Deposits + Time Deposits	
Equity	RCFD3210	1976/03-
Interest Expense, Time	RIAD4174 + RIAD4512	1987/03-1996/12
	RIADA517 + RIADA518	1997/03-2016/12
	RIADHK03 + RIADHK04	2017/03-
Interest Expense, Savings	RIAD4508 + RIAD4509 + RIAD4511	1987/03-
	RIAD4508 + RIAD0093	2001/03-

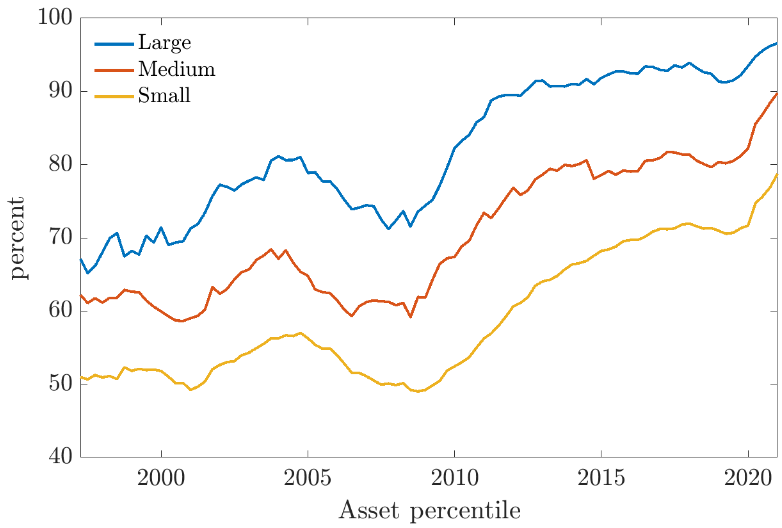
Note: Definition of variables follow Drechsler et al. (2021) except the fact that savings deposits here includes demand deposits and interest expense excludes the deposits in foreign offices.

Table 6: Variable definition and coverage

Ex-post interest rates for savings deposits and time deposits are calculated by interest expense divided by total volume of each deposit products and they are annualized.

F Extra figures and tables

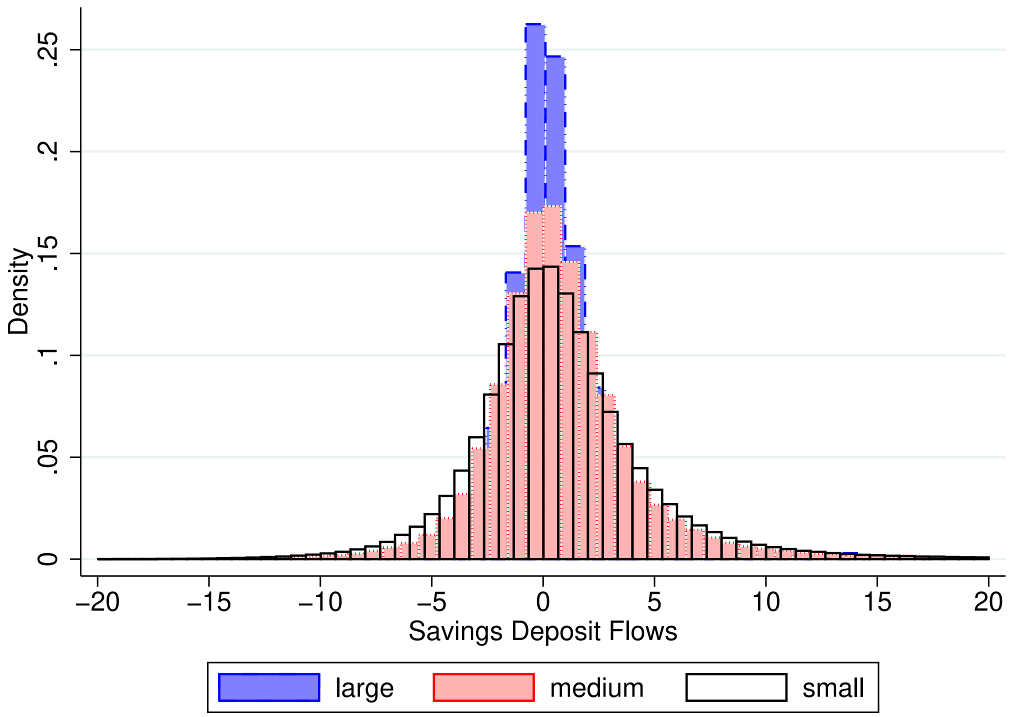
Figure 13 shows a time series of the share of savings deposits in the balance sheet for each size group. This confirms that the time-average pattern provided in Figure 3 consistently holds for each period in the sample.



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1984 Q1 to 2021 Q2. Every aggregation is asset-weighted. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits. *Large* is Top 0.1% banks in asset size. *Medium* is Top 10% excluding Top 0.1% banks. *Small* contains banks in bottom 90% of asset percentile.

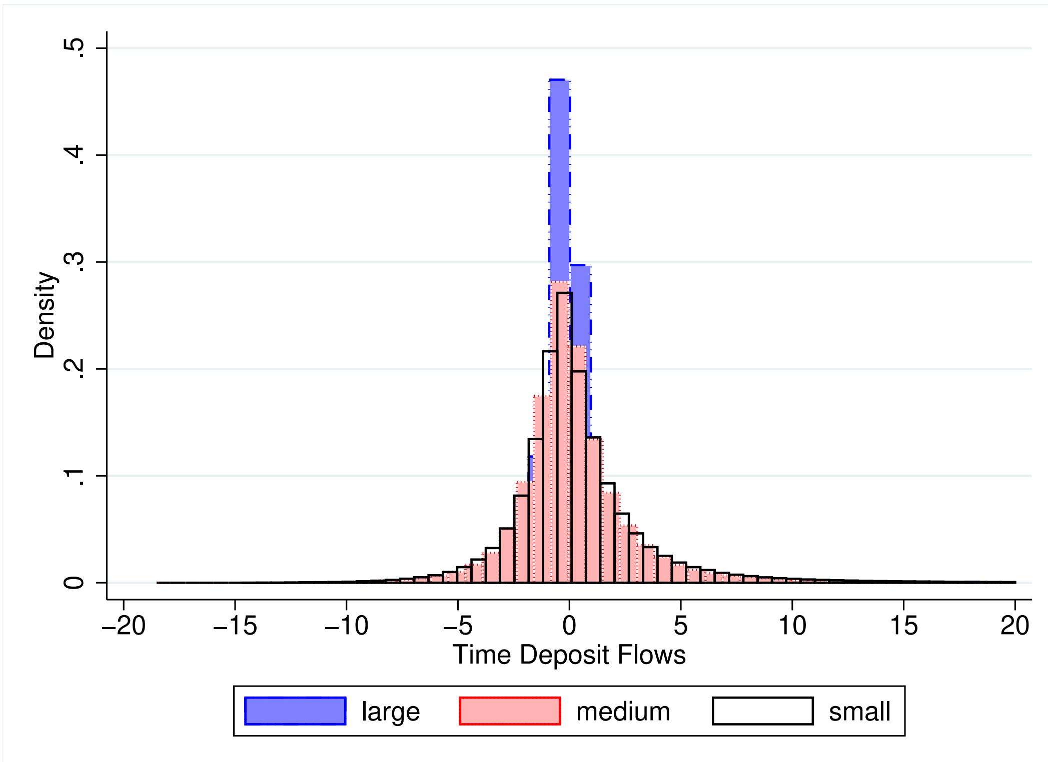
Figure 13: The share of savings deposit, time-series

Figure 14 and 15 display histograms of flows of savings and time deposits, respectively. By comparing between two figures, we see that the flows in savings deposits exhibit a larger variance than time deposits for all size classes. For each type of deposit product, there are clear differences in the frequency of a specific flow. For large banks, both savings and time deposits are relatively more stable and the flows are mostly bounded by the size of 5 percent. However, medium- and small-sized banks are facing large inflows and outflows more often.



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1984 Q1 to 2021 Q2. Deposits used to construct flow variables are deflated with the CPI index. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits. *Large* is Top 0.1% banks in asset size. *Medium* is Top 10% excluding Top 0.1% banks. *Small* contains banks in bottom 90% of asset percentile.

Figure 14: Savings deposit flow, histogram



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1984 Q1 to 2021 Q2. Deposits used to construct flow variables are deflated with the CPI index. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits. *Large* is Top 0.1% banks in asset size. *Medium* is Top 10% excluding Top 0.1% banks. *Small* contains banks in bottom 90% of asset percentile.

Figure 15: Savings deposit flow, histogram